Cryocooler: Theory & Applications

A Thesis Submitted in Partial Fulfillment of the requirement for
The degree of Master of Science in Physics

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February 2008
Dedication

To my parents,

Husband,

Brothers,

Sisters

And all my friends
Acknowledgement

First I thank my God who gave me power to finalize this work.

I would like to acknowledge the great help rendered to me from my supervisor Dr. Omer Ibrahim Eid, without whose encouragement, advises and tolerance, this work would not have been done.

My thanks also extend to the members of Physics Department University of Khartoum. Special thanks to Dr. Marten Deurex from Leiden University (The Netherlands) for his strong support.

I am grateful to the Library of Physics and the main library staff for their assistance.

I thank all my friends who helped me in this work.

Finally, deep gratitude to my husband for his encouragement and patience.
Abstract

In this work we study the Cryocooler (high cooling refrigerators) to describe the current issue of applied physics that working with the theories of thermodynamics (the laws of zero, the first law and the second). The study focused on the 2nd law and its applications such as stirling engine, stirling cycle, carnot cycle, refrigeration cycle, engine heat, heat pump, cooling and stirling types of refrigerators high Cooling (cryocooler) Repertoire (recuperative) and retrospective (regeneration) (regenerative) (working on the repatriation to the first case).

The study also deals with practical experience in the refrigeration in the temperature range (300 - 25K) and the study found a linear relationship between the resistance and Platinum temperature in the range (300 - 25K).

The thesis contains four chapters (introduction, Cryocooler theory and applications, Experimental and setup, the discussion of the results which obtained in this work, and the conclusion).

One of the results of this study is that platinum resistance thermometers have proven reasonable success than using other ordinary thermometers.
The study of liquid nitrogen cooled cryocoolers 


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Chapter one
Introduction

Cryocoolers [1] are the devices used to reach cryogenic temperatures. A cryostat is likely to be used to reach and/or maintain similar conditions or keep some environment in cryogenic stasis.

1.1 Passive coolers [2] are those which require no input power and there are two types in common use:

Radiators. Radiators are panels radiating heat according to Stefan's Law and are the workhorse of satellite cooling due to their extremely high reliability. They have low mass and a lifetime limited only by surface contamination and degradation. It is important; however, to prevent the surface from viewing warm objects and the device must therefore be carefully designed taking the vehicle's orbit into account. They also have severe limitations on the heat load and temperature (typically in the milliwatt range at 70K). Multiple stages are often used to baffle the lowest temperature stage, or 'patch', and it has been shown that efficiency is nearly optimized with three stages, although two is often enough. In this case the first stage consists of a highly reflective baffle (e.g. a cone), to shield the patch from the spacecraft, Earth or shallow-angle sun-light.

Stored cryogenics. Dewar's containing a cryogen such as liquid helium or solid neon may be used to achieve temperatures below those offered by radiators (heat is absorbed by either boiling or sublimation respectively). These provide excellent temperature stability with no exported vibrations but substantially increase the launch mass of the vehicle and limit the lifetime of the mission to the amount of cryogenic stored. They have also proved to be of limited reliability. Passive coolers have been used for many years in space science applications due to their relatively high reliability and low vibration levels, however these are now joined by coolers requiring input power, so called 'active' devices (also termed 'cryocoolers') Active coolers use closed thermodynamic cycles to transport heat up a temperature gradient to achieve lower cold-end temperatures at the cost of electrical input power. The first long-life active cooler system successfully operated in space was a cluster of four rhombic-drives, grease lubricated Sterling cycle machines launched in 1978.
aboard DOD 1-78-1 developed by Phillips and cooling two gamma ray detectors. Although these showed significant performance degradation on-orbit they operated sufficiently well to keep the payload operating until it was destroyed during a successful test of an anti-satellite interceptor in 1985. Since this first generation the flexibility and reliability of active coolers have proved major contributors to the success of many missions.

1.2 The major types of active coolers are:-

Sterling cycle. These coolers are based on causing a working gas to undergo a Sterling cycle which consists of two constant volume processes and two isothermal processes. Devices consist of a compressor pump and a displacer unit with a regenerative heat exchanger, known as a 'regenerator'. Sterling cycle coolers were the first active cooler to be used successfully in space and have proved to be reliable and efficient. Recent years have seen the development of two-stage devices which extend the lower temperature range from 60-80K to 15-30K.

Pulse tube. Pulse tube coolers are similar to the Sterling cycle coolers although the thermodynamic processes are quite different. They consist of a compressor and a fixed regenerator. Since there are no moving parts at the cold-end reliability is theoretically higher than Sterling cycle machines. Efficiencies approaching Sterling cycle coolers can be achieved and several recent missions have demonstrated their usefulness in space.

Joule-Thompson. These coolers work using the well known Joule-Thomson (Joule-Kelvin), effect. A gas is forced through a thermally isolated porous plug or throttle valve by a mechanical compressor unit leading to isenthalpic cooling. Although this is an irreversible process, with correspondingly low efficiency, J-T coolers are simple, reliable, and have low electrical and mechanical noise levels. A J-T stage driven by a valved linear compressor, similar to those used for Sterling cycle and Pulse Tube coolers, will be flown on the planned Planck telescope mission (expected to be launched in 2007).

Reverse Brayton. Reverse/Turbo Brayton coolers have high efficiencies and are practically vibration free. Coolers consist of a rotary compressor, a rotary turbo-
alternator (expander), and a counter flow heat exchanger (as opposed to the regenerator found in Sterling or Pulse Tube coolers). The compressor and expander use high-speed miniature turbines on gas bearings and small machines are thus very difficult to build. They are primarily useful for low temperature experiments (less than 10K), where a large machine is inevitable or for large capacity devices at higher temperatures (although these requirements are quite rare). A recent application of this class of cooler was the Create device used to recover the NICMOS instrument on the Hubble Space Telescope.

The research divided into four chapters, chapter one gives with introduction of the work, chapter two provides the historical background of cryocooler and chapter three describes the cryocooler theory and its applications, chapter four show the discussions, the result and conclusion of this work.

1.3 History of Cryocooler

The first principle of thermodynamics to be established was the second law, as formulated by said Carnot in 1824. By 1860, as found in the works of those as Rudolf Claudius and William Thomson, there were two established "principles" of thermodynamics, the first principle and the second principle. As the years passed, these principles turned into "laws". By 1873, for example, thermodynamics Willard Gibbs, in his "Graphical Methods in the Thermodynamics of Fluids", clearly stated that there were two absolute laws of thermodynamics, a first law and second law. Presently, there are a total of four laws. Over the last 80 years or so, occasionally, various writers have suggested added laws, all of which are from unanimously accepted [1].

1.3.1 The Stirling cycle

The idealized stirling cycle is a thermodynamic cycle with two isochors and two isotherms. Theoretically, it is the most efficient thermodynamic cycle practically possible, however technical issues limit its efficiency when applied- a simpler mechanism is favored over attaining a close fit to the theoretical cycle.
1.3.2 Stirling cryocooler

Any Stirling engine will also work in reverse as a heat pump: i.e. when a motion is applied to the shaft, a temperature difference appears between the reservoirs. One of their modern uses is in refrigeration and cryogenics.

The essential mechanical components of a Striling cryocooler are identical to a Striling engine. The turning of the shaft will compress the working gas causing it temperature to rise. This heat will then be dissipated by pushing the gas against a heat exchanger. Heat would then flow from the gas into this heat exchanger which would probably be cooled by passing a flow of air or other fluid over its exterior. The further turning of the shaft will then expand the working gas. Since it had just been cooled the expansion will reduce its temperature even further. The now very cold gas will be pushed against the other heat exchanger and heat would flow from it into the gas. The external side of this heat exchanger would be inside a thermally insulated compartment such as a refrigerator. This cycle would be repeated once for each turn of the shaft. Heat is in effect pumped out of this compartment, through the working gas of the cryocooler and dumped into the environment. The temperature inside the compartment will drop because its insulation prevents ambient heat from coming in to replace that pumped out.

As with the Striling engine, efficiency is improved by passing the gas through a "Regenerator" which buffers the flow of heat between the hot and cold end of the gas chamber.

The first Striling–cycle cryocooler was developed at Philips in the 1950s and commercialized in such places as liquid nitrogen production plants. The Philips Cryogenics business evolved until it was split off in 1990 to form the Striling Cryogenics & Refrigeration BV. The Netherlands. This company is still active in the development and manufacturing Stirling cryocoolers and cryogenic cooling systems. A wide variety of smaller size cycle in a working gas which is created by high amplitude sound waves.

The thesis contains theoretical background of the cryocooler (high cooling refrigerator) and its most important applications; the discussion of the results obtained in this work is discussed in Chapter four.

In this study using platinum resistance thermometers have proven reasonable success than using other ordinary thermometers.
1.4 Concepts and terminology

Efficiency
The efficiency of a heat engine relates how much useful power is output for a given amount of heat energy input.

Heat pump
A device that warms or cools a building by transferring heat from a relatively low-temperature reservoir to one at a higher temperature.

Heat Engine
Heat engine is defined as a device that converts heat energy into mechanical energy or more exactly a system which operates continuously and only heat and work may pass across its boundaries.
Chapter Two
Cryocooler theory and applications

As we stated earlier in the chapter one definition of cryocooler.

2.1 Types of Cryocoolers:

According to second law of thermodynamics [4], the input power per watt of refrigeration must be at least 2.8W to achieve 80K and 74W to achieve 4K. Small cryocoolers usually operate at 1% to 10% of Carnot efficient, with the higher efficiencies appearing more recently. Figure (2.1) shows schematics of the five common types of cryocoolers used at present. They are divided into two types: recuperative and regenerative.

Building the better cryocooler Recuperative cycle

Figure (2.1): schematics of the five common types of cryocoolers.
2.1.1 Recuperative Cryocoolers

The recuperative cryocooler are analogous to a DC electrical device in the sense that the refrigerant flow is often an advantage because they can transport the refrigerant over fairly large distances to do spot cooling at several locations. The recuperative heat exchangers have two separate flow passage and streams continuously exchange heat with each other. Such heat exchangers are relatively expensive to manufacture. The one-directional flow requires that the compressor has inlet and outlet valves that add complexity and failure points.

The Joule- Thompson (JT) cryocooler cycle is very similar to the vapour-compression cycle used in household refrigerators except for the use of a non-CFC refrigerant to reach cryogenic temperature and effective heat exchanger to span such a large temperature difference. In a domestic refrigerator, oil from the oil-lubricated compressor dissolves in the CFC refrigerant and remains in solution even at the cold end. Since both the CFC and the oil would be frozen at cryogenic temperature, a different refrigerant must be used in cryocooler, and it must be oil free when it enters the main heat exchanger.

Reliable oil free compressors are generally not available at present, although current research may lead to such a device that would also be inexpensive to manufacture. Often oil-lubricated compressors are used with the addition of highly efficient oil – removal equipment in the high-pressure outlet stream.

The irreversible expansion that occurs at the JT valve leads to cooling only for non-ideal gases below some inversion temperature.

Nitrogen gas is typically used for refrigeration at 77K, but the input pressure is usually 20MPa (200bar) to achieve reasonable efficiencies. Hydrogen gas, precooked by a nitrogen stage, is used for refrigeration at 20K, and helium stage is used to achieve the high pressure ratios required. The overall system is very complex with many moving parts to wear out. More often a 4K JT system is precooled to 15-20K with a regenerative refrigerator. A commercial version that produces 1W at 4.2K is available. It is precooled by a Gifford-McMahon refrigerator, requires about 4.5KW of input power; Single –stage JT cooler that use nitrogen or argon with miniature finned-tube heat exchangers have been used in large quantities for rapid (a few second) cool-down of in fared sensors. These open systems use high-pressure gas
from a small storage cylinder. Another type of commercially available open system for 77K operation uses etched glass plates for the heat exchanger and other flow channels. It provides about 0.35W of refrigeration at 77K and above. Reliable compressors for these JT systems have generally not been made in such compressors. The other common recuperative cryocooler is the Brayton cycle refrigerator. An ideal gas such as helium or a helium-neon mixture can be used on this cryocooler because of the reversible expansion that occurs in either the reciprocating or turbo-expenders. As a result, only one fluid is required for all temperatures and much lower pressure ratios are needed. This cycle is commonly used in large liquefaction systems (with a final JT stage) and it has a high reliability due to the use of gas bearings on the turbo-exchanger. This cycle is generally not practical or efficient for refrigeration powers less than 10W at 80K because of the machining problems encountered with such small turbo-exchanger. As a result, its application to the cooling of superconducting electronics is rather limited.

2.1.2 Regenerative Cryocoolers:

The regenerative cryocoolers [5], as shown in Fig. (2.1), uses at least one regenerative heat Exchanger, or regenerator, and operates with oscillating flow and pressure. They are analogous to AC electrical systems, whereas the recuperative cryocoolers are analogous to DC electrical systems. In such an analogy pressure is analogous to voltage, and mass flow or volume flow is analogous to current. Further comparisons with electrical systems will be discussed later. In a regenerator incoming hot gas transfers heat to the matrix of the regenerator, where the heat is stored for a half cycle in the heat capacity of the matrix. In the second half of the cycle the returning cold gas, flowing in the opposite direction through the same channel, picks up heat from the matrix and returns the matrix to its original temperature before the cycle is repeated. At equilibrium one end of the regenerator is at room temperature while the other end is at the cold temperature. Very high surface areas for enhanced heat transfer are easily achieved in regenerators through the use of stacked fine-mesh screen or packed spheres.
2.1.2.1 Regenerative systems

Except for the Gifford-McMahon refrigerator, the compressor or pressure wave generator in regenerative system has no inlet and outlet valves. As a result, it produces an oscillating pressure in the system, and void volumes must be minimized to prevent a reduction in the pressure amplitude. Typical frequencies vary from about 2Hz to gas 60Hz. An oscillating displacer causes the gas to be compressed when it is at the warm end and to expand when it is at the cold end. In the pulse-tube refrigerator, the compressed, hot gas flows from the pulse tube through the warm heat exchanger and the orifice.

The expanded cold gas in the pulse tube flows past the cold heat exchanger when gas from the reservoir returns to the pulse tube. These systems are analogous to AC electrical systems.

The primary heat exchanger is known as a regenerative heat exchanger. It consists of some form of porous material.

1. Stirling Refrigerator

The Stirling cycle, as invented and patented by Robert Stirling in 1815, was first used as a prime mover. In 1834 John Herschel proposed its use as a refrigerator in producing ice. It was not until about 1861 that Alexander Kirk reduced the concept to practice. Air was used as the working fluid in these early regenerative systems. Very little development of Stirling refrigerators [6,7] occurred until 1946 when a Stirling engine at the Philips Company in Holland was run in reverse with a motor and was found to liquefy air on the cold tip [8]. The engine used helium as the working fluid, since earlier work at the company showed helium to give much improved performance to the engines. Research then began on the use of the Stirling refrigerator first as an air liquefier and about ten years later as a cryocooler for cooling infrared sensors to about 80 K. Figure (2.1a) shows the most common form of the Stirling cryocooler used today. Because of its long history, the Stirling cryocooler may be considered the ‘parent’ of the other forms of regenerative cryocoolers shown in Figure (2.1). The Gifford-McMahon and pulse tube refrigerators are variations of the Stirling refrigerator. Operation of the pulse tube refrigerator is best understood by first considering the operating principles of the Stirling refrigerator. The compressor in the Stirling refrigerator is a valve less type; as such, it should be called a pressure
oscillator, but it seldom is. It is simply an oscillating piston, or it could be an oscillating diaphragm. It creates an oscillating pressure in the system where the amplitude of oscillation is typically about 10 to 30% of the average pressure. In order to provide high power densities and keep the system small, the average pressure is typically in the range of 1 to 3 MPa (10 to 30 bar) and frequencies are in the range of 20 to 60 Hz. Helium is almost always used as the working fluid in the regenerative cycles because of its ideal gas properties, its high thermal conductivity, and its high ratio of specific heats. A pressure oscillation by itself in a system would simply cause the temperature to oscillate and produce no refrigeration. The second moving component, the displacer, is required to separate the heating and cooling effects by introducing motion of the gas in the proper phase relationship with the pressure oscillation. When the displacer in Figure (2.1a) is moved downward, the helium gas is displaced to the warm end of the system through the regenerator. The piston in the compressor then compresses the gas, and the heat of compression is removed by heat exchange with the ambient. Next the displacer is moved up to displace the gas through the regenerator to the cold end of the system. The piston then expands the gas, now located at the cold end, and the cooled gas absorbs heat from the system it is cooling before the displacer forces the gas back to the warm end through the regenerator. The correct phasing occurs when the volume variation in the cold expansion space leads the volume variation in the warm compression space by about A 90°. With this condition the mass flow or volume flow through the regenerator is approximately in phase with the pressure. In analogy with AC electrical systems, real power flows only with current and voltage in phase with each other. Without the displacer in the Stirling cycle the mass flow leads the pressure by 90° and no refrigeration occurs. Though the moving piston causes both compression and expansion of the gas, net power input is required to drive the system since the pressure is higher during the compression process. Likewise the moving displacer reversibly extracts net work from the gas at the cold end and transmits it to the warm end where it contributes some to the compression work. In an ideal system, with isothermal compression and expansion and a perfect regenerator, the process is reversible. Thus, the coefficient of performance COP for the Stirling refrigerator is the same as the Carnot COP given by

$$\text{COP}_{\text{carnot}} = \frac{Q_c}{W_0} = \frac{T}{T_h - T_c}$$ (2.1)
Where \( Q_C \) is the refrigeration power, \( W_0 \) is the power input, \( T_c \) is the cold temperature and \( T_h \) is the hot temperature. The occurrence of \( T_c \) in the denominator arises from the PV power (proportional to \( T_c \)) recovered by the expansion process and used to help with the compression.

2. Gifford-McMahon Refrigerator

Because the pressure oscillates everywhere within the Stirling refrigerator [5], excess void volumes must be minimized to maintain a large pressure amplitude for a given swept volume of the piston. Thus, oil removal equipment cannot be tolerated, which means that the moving piston and displacer must be oil-free. But long lifetimes then become difficult to achieve. In the mid-1960s Gifford and McMahon [9] showed that the pressure oscillation for cryocoolers could be generated by the use of a rotary valve that switches between high- and low-pressure sources. The Gifford-McMahon refrigerator, shown in Figure (2.1c) has the same low-temperature parts as the Stirling refrigerator. The irreversible expansion through the valves significantly reduces the efficiency of the process, but the advantage of this approach is that it allows for an oil-lubricated compressor with oil-removal equipment on the high side to supply the high- and low-pressure sources. Oil-lubricated compressors were readily available at low cost from the air-conditioning industry by the mid-1960s, and they had lifetimes of at least 5 years with continuous operation. To maintain a 1 to 3 year lifetime for the PTFE-based seals on the displacer and the rotary valve, Gifford and McMahon used low speeds of 1 to 2 Hz for those two components in the cold head. The cold head could be placed quite some distance from the compressor and connected by flexible lines for the high- and low-pressure gas. These Gifford-McMahon refrigerators are now manufactured at a rate of about 20,000 per year for use in cryopumps.

2.2 Efficiencies of Real Pulse Tube Refrigerators

Figure (2.2) shows a comparison of the efficiency (in terms of % of Carnot) of actual pulse tube refrigerators that have achieved high efficiencies. The efficiencies reported here refer to the input electrical power to the compressor. In a few cases the number 85 associated with a data point means that the efficiency was based on input PV power that was divided by 0.85 to obtain the electrical input power if an 85% efficient compressor had been used. The majority of pulse tube refrigerators reported in the
literature has not achieved efficiencies anywhere near these values. Careful attention to details in the design of these high efficiency refrigerators is required and expensive experimental optimization is often required even after the most careful computer modeling and optimizations are performed. In most cases these detailed designs remain proprietary information. Shown for comparison are data for recent, high efficiency Stirling, Gifford-McMahon, Brayton, and mixed-refrigerant Joule-Thomson refrigerators, all operating at temperatures near 80 K. In Figure (2.2) shows a general trend in all refrigerators for increased efficiency as the size increases. The graph also indicates that pulse tube refrigerators have equaled or exceeded the efficiency of the best Stirling refrigerators. As a result, pulse tube refrigerators have now become the most efficient cryocoolers for a given size. Efficiencies as high as 17% of Carnot are now possible when using an 85% efficient compressor. Most of the data points referring to electrical input actually have a compressor that is about 85% efficient when a valve less compressor is used. As expected, the use of a valve compressor reduces the efficiency of the pulse tube refrigerator to about that of Gifford-McMahon refrigerators.

The earliest pulse tube refrigerator to achieve these high efficiencies was the large NIST pulse tube in 1991 that achieved 31 W at 80 K with 602 W input PV power at 316 K (13% of Carnot assuming an 85% efficient compressor). Within about a year the mini pulse tube refrigerator discussed earlier achieved about 8% of Carnot. Most of the other pulse tube refrigerators with high efficiencies have been developed in the last few years for space applications. The system with 17% efficiency at 222 W of input power is the NIST oxygen liquefier described below. It operates at 45 Hz and uses a double-diameter inertance tube for increased phase shift [10].

![Figure (2.2):- Carnot efficiency of various cryocooler types Data taken from[2]](image-url)
2.3 Applications and Cryocooler Requirements

Table (2.1) lists the major applications of Cryocoolers [5] that are currently in use or have some potential for large impacts. For many years the largest application for cryocoolers was for use by the military in cooling infrared sensors to about 80 K for tactical applications in tanks, airplanes, and missiles. Since about 1950 over 100,000 cryocoolers have been manufactured in the U. S. alone for this application. Refrigeration powers range from about 0.15 W to about 2 W. Stirling cryocoolers, used primarily for this application in the last twenty years, have been able to meet the requirements reasonably well. However, their mean-time-to-failure (MTTF) of about 4000 hours (0.5 years) is far short of the required 5 to 10 year lifetime needed for satellite applications or even the 3 to 5 year lifetime required for most commercial applications. The rapid growth of research and development on pulse tube refrigerators in the last ten years has occurred because of its potential for improved reliability, lower vibration, and lower cost. This research and development has also led to very high efficiencies.

In the past ten years the largest commercial application of Cryocoolers has been for Cryopumps (about 20,000/year) for the semiconductor fabrication industry. These cryopumps require a few watts of refrigeration at a temperature of about 15 K to cool a charcoal adsorbent bed and a few tens of watts at about 80 K to cryopump mostly water vapor. A two-stage Gifford-McMahon (GM) refrigerator has been used for nearly all of these cryopumps. In the past few years the vibration of the moving displacer in the GM refrigerator has become a problem in semiconductor fabrication as the circuit line widths become narrower. Two-stage pulse tube refrigerators are now being studied in several laboratories for this application. Two stage pulse tube refrigerators are capable of reaching temperatures down to about 2 K [11].
<table>
<thead>
<tr>
<th>Category</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Military</td>
<td>1. Infrared sensors for missile guidance</td>
</tr>
<tr>
<td></td>
<td>2. Infrared sensors for surveillance (satellite based)</td>
</tr>
<tr>
<td>Police and Security</td>
<td>1. Infrared sensors for night vision and rescue</td>
</tr>
<tr>
<td>Environmental</td>
<td>1. Infrared sensors for atmospheric studies of ozone hole and greenhouse effects</td>
</tr>
<tr>
<td></td>
<td>2. Infrared sensors for pollution monitoring</td>
</tr>
<tr>
<td>Commercial</td>
<td>1. Cryopumps for semiconductor fabrication</td>
</tr>
<tr>
<td></td>
<td>2. High temperature superconductors for cellular-phone base stations</td>
</tr>
<tr>
<td></td>
<td>3. Superconductors for voltage standards</td>
</tr>
<tr>
<td></td>
<td>4. Semiconductors for high speed computers</td>
</tr>
<tr>
<td></td>
<td>5. Infrared sensors for NDE and process monitoring</td>
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<tr>
<td>Medical</td>
<td>1. Cooling superconducting magnets for MRI systems</td>
</tr>
<tr>
<td></td>
<td>2. SQUID (superconducting quantum interference device) magnetometers for heart and brain studies</td>
</tr>
<tr>
<td></td>
<td>3. Liquefaction of oxygen for storage at hospitals and home use</td>
</tr>
<tr>
<td>Energy</td>
<td>1. LNG for peak shaving</td>
</tr>
<tr>
<td></td>
<td>2. Infrared sensors for thermal loss measurements</td>
</tr>
<tr>
<td></td>
<td>3. Supercond. Mag. energy storage for peak shaving and power conditioning</td>
</tr>
</tbody>
</table>
Table (2.2): Advantages/disadvantages of different types of cooler technology (table shows data taken from [2])

Usually the coolers can be run at lower or higher temperatures with correspondingly lower or high heat lifts.

<table>
<thead>
<tr>
<th>Cooler</th>
<th>Typical temperature</th>
<th>Typical heat lift</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cryogen</td>
<td>4 K</td>
<td>0.05 W</td>
<td>Stable, low vibration</td>
<td>Short lifetime, out-gassing,</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>massive, complex</td>
</tr>
<tr>
<td>Stirling -- 1 stage</td>
<td>80 K</td>
<td>0.8 W</td>
<td>Efficient, heritage</td>
<td>Vibrations</td>
</tr>
<tr>
<td>Stirling -- 2 stage</td>
<td>20 K</td>
<td>0.06 W</td>
<td>Intermediate temp</td>
<td>Under development</td>
</tr>
<tr>
<td>Pulse tube</td>
<td>80 K</td>
<td>0.8 W</td>
<td>Lower vibrations</td>
<td>Lower efficiency than Stirling</td>
</tr>
<tr>
<td>Joule-Thompson</td>
<td>4 K</td>
<td>0.01 W</td>
<td>Low vibrations</td>
<td>Requires hybrid design</td>
</tr>
<tr>
<td>Rev. Brayton</td>
<td>65 K</td>
<td>8 W</td>
<td>High capacity</td>
<td>Complex</td>
</tr>
</tbody>
</table>
Chapter Three
Experimental and Setup
In this chapter, we will present some results obtained from experiments conducted at the Physics Department, U. of K.

3.1 Materials and method
The cryogenic facility
In accordance with the company [12] which made it we call the apparatus a cryogenerator (a machine that generates cold).

In short the cryogenerator consists of two parts:
* A compressor, at room temperature, in which He gas is compressed to a pressure of 25 Bar.
* A "cold heat" in which the gas is made to expand after expansion, the gas flows back to the compressor where it is compressed again. and so on the "cold" is produced by the expansion of the gas in the cold heat. Saying it in a different way, during expansion the circulating helium gas takes in heat from a "platform" in the cold heat on which samples to be measured are located.

The cycle through which the helium gas:
Compression (heat given away to cooling water in the compressor
1. Transfer of gas to the cold heat.
2. Expansion (heat taken in from the platform.
3. Transfer of the gas back to the room temperature.

Compressor is called the Gifford. Mc Mahon cycle
"Gifford –Mc Mahon cryogenerators "consist of"
1. The compressor unit
2. The flexi lines

A description of the physics and technique of the Gifford – Mc Mahon cycle, is much analogue to the Stirling cycle

3.2 The Refrigerator
We have seen that a heat engine is a device [13] by which a system is taken through a cycle in such a direction that some heat is absorbed while the temperature is high, a smaller amount is rejected at lower temperature, and a net amount of work is done on
the outside. If we imagine a cycle performed in a direction opposite to that an engine, the rejection of a larger amount at a higher temperature, and a net amount of work done on the system. A device that performs a cycle in this direction is called refrigerator, and the system undergoing the cycle is called a refrigerant.

The Stirling cycle is capable of being reversed and, when reversed, it gives rise to one of the most useful types of refrigerator. The operation of an ideal Stirling refrigerator may best be understood with the aid of the schematic diagrams shown in Fig.2.1a, and the accompanying PV diagram of Fig. 2.1b.

1→2 while the right piston remains stationary, the left piston moves up, compressing the gas isothermally at the temperature $\theta_H$ and rejecting heat $Q_H$ to the hot reservoir.

2→3 both pistons move the same amount simultaneously, forcing gas through the regenerator, giving up some heat $Q_R$ to the regenerator, and emerging cold in the right-hand space. This takes place at constant volume.

3→4 while the left piston remains stationary [14], the right piston moves down and causes an isothermal expansion at the low temperature $\theta_C$, during which heat $Q_C$ is absorbed by the gas from the cold reservoir.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{stirling_cycle_diagram.png}
\caption{Schematic diagram of steps in the operation of idealized Stirling refrigerator}
\end{figure}
Both pistons move and force gas at constant volume from the cold to the hot end through the regenerator, thereby taking up approximately the same heat $Q_R$ that was supplied to the regenerator in process $2\rightarrow 3$.

Figure (3.2): Idealized Stirling refrigeration cycle on PV diagram

3.3 Sample preparation

This section deals with the specific form and structure of the sample in so far as these are important for handing the samples.

In the preparation of the experiment, some electrical connections were made. The thin copper wires were attached to the sample. The samples were placed on the platform in the cryogenerator.

Figure (3.3): Sample on the platform in the cryogenerator

There are two options for connecting the leads to the gold contacts
1- Soldering by just touching the gold contacts with small soldering iron (Christianne Beekman).
2- Attachment of leads to gold contacts with silver paint
Preferred using the silver paint (Leiden, Amsterdam).
Figure (3.4): 2-Wires and 4-Wires resistance measurement

Determine $R_1 + R_2$ separately, can be used when lead resistances $R_1 + R_2$ are small and the temperature change predictable.
3.4 Cryocooler circuit and components

\[ V_{\text{output}} = 3V \]

\[ R_c = 1000\Omega \]

\[ V_{\text{output}} = V_{\text{constant}} + V_{\text{variable}} \]

\[ V = IR \]

\[ \frac{V_c}{V} = \frac{R_v}{R_c} \]

\[ \frac{V_c}{R_c} = \frac{V_c}{R_c} \times \frac{R_c}{(V_{\text{total}} - V_v)} \]

Figure (3.5): This scheme for device of cryocooler and circuit from to connect
3.5 Components of cryocooler

Figure (3.6): Cryogenics: Cryogenics is generally the science used to produce very low-temperatures and study the characteristics of substances at these low temperatures.

Figure (3.7): Sample thermometer

Figure (3.8): Sample Pt
Figure (3.9): Vacuum Pump

Figure (3.10): Compressor: The compressor is the heart of the cryo system

Figure (3.11): Connector

Figure (3.12): Water tape and Drain Water Cooler
Chapter four

Results and Discussion

4.1 Lakeshore (gives the expected R vs T): Sensor type: platinum resistor

**Table (4.1):** Measurement and control technologies Calibration report No.197015
Sensor Type: Platinum Resistor

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Resistance (ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>299.937</td>
<td>110.472</td>
</tr>
<tr>
<td>309.955</td>
<td>114.349</td>
</tr>
<tr>
<td>289.946</td>
<td>106.598</td>
</tr>
<tr>
<td>279.968</td>
<td>102.708</td>
</tr>
<tr>
<td>259.955</td>
<td>94.8787</td>
</tr>
<tr>
<td>229.929</td>
<td>83.0404</td>
</tr>
<tr>
<td>219.945</td>
<td>79.0771</td>
</tr>
<tr>
<td>209.946</td>
<td>75.0934</td>
</tr>
<tr>
<td>179.948</td>
<td>63.0529</td>
</tr>
<tr>
<td>169.954</td>
<td>59.0067</td>
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<td>149.941</td>
<td>50.8466</td>
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<tr>
<td>99.9269</td>
<td>30.0099</td>
</tr>
<tr>
<td>25.7769</td>
<td>2.74567</td>
</tr>
</tbody>
</table>
The measurement of the resistance of the sample Christine who sent. I have added a figure of an R-T measurements here the metal + insulator transition can clearly be seen the sample is at high T an insulator and at low T a ferromagnetic metal. Normally we do measurements DC with a low current 0.1 μA to 1 μA. Then there is no heating effect.

But, the resistances of these samples do change with change currents. This is so because magnetic moments can be oriented by the current. The resistance decreases at increasing current.
4.2 Tests's reading of experimental relationship between resistance Pt and low temperature

Table (3.3) Pt thermometer R (Pt100) in experimental vs T (temperature)

<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>Resistance (ohm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>1107</td>
</tr>
<tr>
<td>298</td>
<td>1095</td>
</tr>
<tr>
<td>290</td>
<td>1058</td>
</tr>
<tr>
<td>275</td>
<td>985</td>
</tr>
<tr>
<td>253</td>
<td>936</td>
</tr>
<tr>
<td>222</td>
<td>802</td>
</tr>
<tr>
<td>207</td>
<td>735</td>
</tr>
<tr>
<td>173</td>
<td>621</td>
</tr>
<tr>
<td>150</td>
<td>499</td>
</tr>
<tr>
<td>100</td>
<td>299</td>
</tr>
<tr>
<td>25</td>
<td>29</td>
</tr>
</tbody>
</table>
Figure (4.2): Calibration Resistance vs Temperature from experimental
Conclusion

In this study the cryocooler applications, types and components are described. In this project a platinum resistance thermometer in an electric circuit is utilized in constructing a cryocooler. The platinum resistance thermometer is used to specify the temperature by measuring the corresponding variation in resistance. Resistance thermometers have proven reasonable success when used with very low temperatures (300-25k) where it is not possible to use other ordinary thermometers. Because of its purity and relatively high conductivity, Platinum has used in this project to calibrate the cryocooler starting from 300 K. The minimum temperature reached was 25K with a corresponding resistance of 29 Ohm as shown in figures (4-1), (4-2), where a 4 Ohm decrease in resistance approximately corresponds to 1 Kelvin decrease in temperature. The minimum temperature supposed to be reached by the cryocooler is A 4 K but a leakage in the machine prevent us from reaching this temperature. The results found in this study could be used as reference data for the apparatus to show that using platinum resistance is reasonably acceptable.
Appendix

Theoretical Background of Thermodynamics

A.1 Thermal physics

Michael and O’Sullivan [15] suggested six aims to thermal physics:
to introduce the ideal gas and the thermodynamic temperature scales,
to investigate the connection between the amount of mechanical energy generated by
friction in a body and the corresponding change in its temperature,
to study the phenomena of thermal conduction, convection, radiation and expansion,
to develop a formalism (called thermodynamic) to describe the macroscopic behavior
of systems when their temperature change and, simultaneously, mechanical work is
done on or by the surrounding,
to study transformations (fusion and vaporization) between different phases of matter, and
and
to interpret thermodynamic behavior in terms of microscopic models.

A.2 Development of classical thermodynamics

In 1824, Carnot* introduced several new concepts. The first was the cycle [16], the
process which left no net change in the condition of a system. This was the first
instance of consistent use in a thermodynamic argument of the quantity which is heat
in the modern sense of the term. Many years were to elapse before this usage was to
become established.
The principle of thermodynamics to be established was the second law [17], as
formulated by sadi Carnot in 1824. By 1860, as found in the works of those as Rudolf
Clausisus and William Thomson, there were two established "principle " of
thermodynamics, the first principle and the second principle. As the years passed,
these principles turned into "laws". By 1873, for example, thermodynamics Willard
Gibbs, in his "Graphical Methods in the thermodynamics of fluids" clearly stated that
there were two absolute laws of thermodynamics, a first law and a second law.
Presently these are a total of four laws. Over the last 80 years or so, occasionally,
various writers have suggested added laws, all of which are far from unanimously
accepted.

*Nicolas Léonard Sadi Carnot (June 1, 1796-August 24, 1832) was a French physicist
and military engineer who gave the first successful theoretical account of heat
engines, now known as the Carnot cycle, thereby laying the foundations of the second law of thermodynamics. Technically, he is the world's first thermodynamics, being responsible for such concepts as Carnot efficiency, Carnot theorem, Carnot heat engine, and others.

A.3 Laws of thermodynamics

The laws of thermodynamics [18], in principle, describe the specifics for the transport of heat and work in thermodynamic processes. Since their conception, however, these laws have become some of the most important in all of physics and other branches of science connected to thermodynamics [17].

A.3.1 Zeroth law of thermodynamics

As we talked earlier in chapter one about definition zeroth law of thermodynamics and relationship of thermal equilibrium.

![Figure (A.1): thermal equilibrium](image)

If A and B are in thermal equilibrium with C, then A is in thermal equilibrium with B. Practically this means that all three are at the same temperature and it forms the basis for comparison of temperatures. It is so named because it logically precedes the First and Second Laws of Thermodynamics.

A.3.1.1 Thermal Equilibrium

It is observed that a higher temperature object which is in contact with a lower temperature object will transfer heat to the lower temperature object. The objects will approach the same temperature, and in the absence of loss to other objects, they will then maintain a constant temperature. They are then said to be in thermal equilibrium. Thermal equilibrium is the subject of the Zeroth Law of Thermodynamics.
A.3.1.2 Temperature and the zeroth law

It is often claimed, for instance by Max Planck in his influential textbook on thermodynamics, that this law proves that we can define a temperature function, or more informally, that we can `construct a thermometer`. Whether this is true is a subject in the philosophy of thermal and statistical physics.

In the space of thermodynamic parameters, zones of constant temperature will form a surface, which provides a natural order nearby surface; it is then simple to construct a global temperature function that provides a continuous ordering of states. Note that the dimensionality of a surface constant temperature is one less than the number of thermodynamic parameters (thus, for an ideal gas described with 3 thermodynamic parameter P, V and n, they are 2D surface) the temperature so defined may indeed not look like the Celsius temperature scale, but it is a temperature function.

For example, if two systems of ideal gas are in equilibrium, then

\[
\frac{P_1 V_1}{N_1} = \frac{P_2 V_2}{N_2}
\]

Where \( P_i \) is the pressure in the \( i^{th} \) system

\( V_i \) is the volume, \( N_i \) is the amount

(In moles, or simply number of atoms) of gas.

The surface \( PV/N = \text{const} \) defines surfaces of equal temperature, and the obvious (but not only) way to label them is to define \( T \) so that \( PV/N = RT \) where \( R \) is some constant. These systems can now be used as a thermometer to calibrate other systems.

A.3.2 First Law of Thermodynamics

The first law of thermodynamics is the application of the conservation of energy principle to heat and thermodynamic processes: The change in internal energy of a system is equal to the heat added to the system minus the work done by the system.

\[
\Delta U = Q - W
\]

\( \Delta U \) \equiv change in internal energy

\( Q \) \equiv heat added to the system

\( W \) \equiv work done by the system
The first law makes use of the key concepts of internal energy, heat, and system work. It is used extensively in the discussion of heat engines. It is typical for chemistry texts to write the first law as equation (2-2) it is the same law, of course - the thermodynamic expression of the conservation of energy principle. It is just that W is defined as the work done on the system instead of work done by the system. In the context of physics, the common scenario is one of adding heat to a volume of gas and using the expansion of that gas to do work, as in the pushing down of a piston in an internal combustion engine. In the context of chemical reactions and process, it may be more common to deal with situations where work is done on the system rather than by it.

A.3.2.1 Heat Transfer

The transfer of heat is normally from a higher temperature object to a lower temperature object. Heat transfer changes the internal energy of both systems involved according to the First Law of Thermodynamics.

A.3.2.2 Internal Energy and the First Law of Thermodynamics

The total energy possessed [19] by all the particles of a gas is called the internal energy of the gas, U. If work is done, the internal energy can be changed. If the gas is in an insulating container, the change in internal energy is equal to the work done. If the gas is in a conducting container then energy, can enter or leave the gas. If no work is done, The process of changing the internal energy of a gas without doing work is called heating* (or cooling) the gas.

Combining the two statements gives the first law of thermodynamics, usually stated as equation (2-2). This law is a thermodynamic statement of the law of conservation of energy. A useful form of the equation is

\[
\Delta U = P \Delta V + Q
\]

If Q entering the gas is positive, work done on the gas is positive.
A.3.3 Second Law of Thermodynamics

The second law of thermodynamics is a general principle which places constraints upon the direction of heat transfer the attainable efficiencies of heat engines. In so doing, it goes beyond the limitations imposed by the first law of thermodynamics. Its implications may be visualized in terms of the waterfall analogy.

*Notice that in thermodynamics we do not have a use for the word "heat" as a noun only as a verb (hence the title chosen for another part of the thermal physics section).

**Figure (A .2): The maximum efficiency which can be achieved is the Carnot efficiency.**

Second Law of Thermodynamics: It is not possible for heat to flow from a colder body to a warmer body without any work having been done to accomplish this flow. Energy will not flow spontaneously from a low temperature object to a higher temperature object. This precludes a perfect refrigerator. The statements about refrigerators apply to air conditioners and heat pumps, which embody the same principles.

This is the "second form" or Clausius statement of the second law.

**Figure (A .3): Clausius statement of the second law.**
A.3.3.1 Heat Flow to Hotter Region

Although internal energy will not spontaneously flow from a cold region to a hot region, it can be forced to do so by doing work on the system. Refrigerators and heat pumps are examples of heat engines which cause energy to be transferred from a cold area to a hot area [19]. Usually this is done with the aid of a phase change, i.e., a refrigerant liquid is forced to evaporate and extract energy from the cold area. Then it is compressed and forced to condense in the hot area, dumping its heat of vaporization into the hot area.

![Figure (A.4): a refrigerant liquid](image)

A.3.3.2 Heat engines

It is a device that enables us to take energy from a heat reservoir at a higher temperature [15]. (T2) and depositing a smaller amount of energy at lower temperature (T1). If the process is reversed i.e. A→D→C→B→A) work is put into the systems, the result is (In figure (2-6)) that heat energy is taken from the low temperature reservoir and deposited in a higher temperature reservoir i.e. the device is heat pump [20] or refrigerator.

![Figure (A.5): Heat pump](image)
A.3.3.2.1 Refrigerator

A refrigerator is a heat engine in which work is done on a refrigerant substance in order to collect energy from a cold region and exhaust it in a higher temperature backdrop, thereby further cooling the cold region.

Refrigerators have made use of fluorinated hydrocarbons with trade names like Freon-12, Freon-22, etc. which can be forced to evaporate and then condense by successively lowering and raising the pressure. They can therefore "pump" energy from a cold region to a hotter region by extracting the heat of vaporization from the cold region and dumping it in the hotter region outside the refrigerator. The statements about refrigerators apply to air conditioners and heat pumps, which embody the same principles.
Although this process works very well and has been in place for decades, the bad news about it is that fluorinated hydrocarbons released into the atmosphere are potent agents for the destruction of the ozone in the upper atmosphere. Therefore tighter and tighter restrictions are being placed on their use.

Figure (A.8): Components of a refrigerator (from 2006 Merriam-Webster, Ino)

Components of a refrigerator. A compressor pressurizes the refrigerant gas, heating it and forcing it through the system. The gas cools and liquefies in the condenser, giving up its heat to the outside air. The liquid's pressure is lowered when it passes through an expansion valve, and there is an associated further drop in temperature. The cold liquid then passes into the evaporator coils, where heat drawn from the warmer refrigerator compartment causes it to vaporize. The gas is then returned to the compressor to repeat the cycle.
In the tubes around the freezer compartment, the pressure is decreased by the pump (there is a small section of the tube which is narrower than the rest). Rapid evaporation takes place here and latent heat of vaporization is taken in.

In the tubes outside the refrigerator, the vapour is compressed and then it condenses. Latent heat is given out as it condenses.

**A.3.3.2.2 Heat Pumps**

If a heat engine is operated in reverse, as described above, it has the effect of transferring internal energy from a body at a low temperature to one at a higher temperature [19]. It is then called a “heat pump” as see fig. (2-5) (or a refrigerator depending on what it is used for).

A heat pump or fridge can be represented by a similar diagram to the one used for the heat engine but with the arrows representing energy flows reversed.

An explanation of the operation of a fridge requires consideration of cooling [16] caused by evaporation.

The temperature of a body is a measure of the average kinetic energy of its particles. During evaporation, the molecules which are more likely to "escape" from liquid and become part of the vapour are the ones which have higher than average kinetic energy. Therefore, if you cause the rate of evaporation of a liquid to increase, without supplying energy, the temperature of the remaining liquid will decrease.

The rate of evaporation of a liquid can be increased by

*Decreasing the pressure acting on its surface
* blowing air over the surface (clothes dry more quickly on a windy day)
* increasing the surface area of the liquid (evaporation only occurs at the surface)
* Increasing the temperature

### A.3.3.3 The Carnot Cycle

The sequence below (called the Carnot cycle) is not supposed to describe a practical engine [19]. Carnot was attempting to find the theoretical maximum for the efficiency of a heat engine.

Consider a quantity of gas in a cylinder having a piston which can move without friction. The walls of the cylinder are perfect insulators except at the base.

![Figure (A.10): The Carnot Cycle](image)

1. Gas at temperature $T_H$
2. Gas at temperature $T_H$ placed in contact with heat source at temperature $T_H$.
3. Isothermal expansion at $T_H$ (slow expansion). Work done by the gas.
4. Gas removed from source and insulated.
5. Adiabatic expansion until temperature reaches $T_C$. Work done by the gas.
6. Gas at temperature $T_C$ placed in contact with heat sink at temperature $T_C$.

Back to 1…and so on, and so on…
The expansions are done at high temperature and the compressions are done at lower temperature. Therefore, the work done by the gas is greater than the work done on the gas.

The net effect of the cycle is that a quantity of work, \( w = Q_H - Q_C \) has been done and a quantity of thermal energy \( Q_C \) has been transferred from a hot body (the source) to a cold body (the sink).

The cycle is reversible. This means that if the sequence is followed in the reverse order the net result will be that an external agent does a quantity of work, \( w \), transferring a quantity of internal energy, \( Q \), from the sink to the source.

It was stated that there was no friction and that the internal energy transfers took place under conditions very close to thermal equilibrium (this is because they took place very slowly, in a perfectly conducting container). If there is friction and/or if the energy transfers take place far from equilibrium (rapid compression and expansion) the cycle will not be reversible (in the sense defined above).

After considering this cycle of operations Carnot proposed the following theorem, now called Carnot’s theorem.

The maximum efficiency of a heat engine depends on the difference in temperature between source and sink and for given source and sink temperatures the most efficient engine is a reversible engine.

A.3.3.3.1 Entropy and the Carnot Cycle

![Figure (A.11): Entropy and the Carnot Cycle](image)
These equations are explain relationship between Carnot efficiency and Entropy.

The efficiency of a heat engine cycle is given by

\[ \eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} \]

For the ideal case of the Carnot cycle, this efficiency can be written

\[ \eta = \frac{T_H - T_C}{T_H} \]

Using these two expressions together

\[ 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H} \]

\[ \frac{Q_C}{T_C} = \frac{Q_H}{T_H} \text{ or } \frac{Q_H}{T_H} - \frac{Q_C}{T_C} = 0 \]

If we take Q to represent heat added to the system, and then heat taken from the system will have a negative value. For the Carnot cycle [19]

\[ \sum Q_i \frac{1}{T_i} \]

This can be generalized as an integral around a reversible cycle

Clausius Theorem

\[ \int \frac{dQ}{T} = 0 \]

For any part of the heat engine cycle, this can be used to define a change in entropy S for the system

\[ S(B) - S(A) = \int_A^B \frac{dQ}{T} \]
Or in differential form at any point in the cycle

\[ d\mathcal{S} = \frac{dQ}{T} \]

For any irreversible process, the efficiency is less than that of the Carnot cycle. This can be associated with less heat flow to the system and/or more heat flow out of the system. The inevitable result is

\[ \sum_i \frac{Q_i}{T_i} = 0 \]

Clausius Inequality

\[ \Phi \frac{dQ}{T} \leq 0 \]

Any real engine cycle will result in more entropy given to the environment than was taken from it, leading to an overall net increase in entropy.

A.4 PV Diagrams

A "PV diagram" Figure (2-11) showing changes in the pressure and volume of a sample of gas.

It is useful to be able to recognize various types of change of the state of a gas from a PV diagram [19]. Four examples are given below.

![Figure (A.12): PV diagram](image)

1. Change of p (and T) at constant volume; an is volumetric change.
2. Change of V (and T) at constant pressure; an isobaric change.
3. Change in p and V at constant temperature; an isothermal change.
4. Change in p and V in an insulated container (no heating of the gas); an adiabatic change.
In practice, changes of state do not quite follow any of these ideal paths. However, approximate isothermal, adiabatic etc changes can occur.

In practice: For an isolumetric change, heat the gas in a fixed volume container (one made of a material having a low thermal expansively)

For an isobaric change, trap a small quantity of the gas in a tube using a thread of mercury and heat it slowly for an isothermal change, compress (or expand) the gas slowly in a container of high thermal conductivity for an adiabatic change, compress (or expand) the gas rapidly in a container of low thermal conductivity [19].

On a PV diagram, an isothermal looks pretty much like an adiabatic. However, if an isothermal and an adiabatic have a point in common, then the adiabatic is the curve having the greater slope at that point

A.5 The Thermodynamic Identity

A useful summary relationship called the thermodynamic identity makes use of the power of calculus and particularly partial derivatives. It may be applied to examine processes in which one or more state variables are held constant, e.g., constant volume, constant pressure, etc. The thermodynamic identity holds true for any infinitesimal change in a system so long at the pressure and temperature are well defined. It is presumed that the number of particles is constant (i.e., you are dealing with the same system before and after the change).

Thermodynamic identity:

\[ dU = TdS - PdV \]

"d" denotes the total differential of the associated quantity

U = internal energy
S = entropy
V = volume
T = temperature
P = pressure

A.5.1 Pressure and the Thermodynamic Identity

Thermodynamic identity can be used to obtain a relationship between pressure and entropy.
If the internal energy is held constant, then the pressure can be expressed as

\[ p = T \left( \frac{\partial s}{\partial V} \right)_{U,N} \]

This definition implies that you are holding both the internal energy and the number of particles constant in taking the derivative. This can be applied to the expression for the entropy of a monatomic ideal gas

\[ S_{\text{ideal, gas}} = Nk \left[ \ln \left( \frac{V}{N} \frac{3}{3NkT} \right)^{3/2} \right] \]

Consider the partial derivative of this Equ (2-14) with respect to V to get the pressure, this gives us:

\[ P = T \frac{\partial}{\partial V} (NK \ln V) = \frac{NKT}{V} \]

\[ PV = NKT \]

This relationship is just the ideal gas law! But the ideal gas law can be obtained from just Newton's laws, which give an expression for average pressure of gas. That along with the kinetic temperature gives the form of the gas law. So, looking at it from the other direction, the ideal gas law offers confirmation of the relationship between pressure and entropy.
References


[17] www.cryocooler\system.htm

