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1 AUTHOR:

Arbab I. Arbab
University of Khartoum

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ORIGINAL ARTICLE

On the gravitational radiation of gravitating objects

Arbab I. Arbab

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Abstract In the framework of unifying gravity and electromagnetism, we have shown that accelerating objects emit gravitational wave as those determined by Larmor formula for the accelerating charged particle. We have found new formulae for the power of Gravitational waves radiated by spinning and orbiting objects. The minimum wavelength of the gravitational wave emitted by an object of mass m and radius R is $\lambda_{\rm min.}=\sqrt[4]{\frac{32}{3}}\sqrt{\frac{\pi^2 GmR}{c^2}}$.

Keywords Gravitational waves · Unification of forces · Electromagnetism · Larmor radiation

1 Introduction

In 1916 Albert Einstein predicted based on General Relativity that vibrating (accelerated) masses should create gravitational waves. But because of the weak strength of gravity, only enormous masses undergoing huge accelerations would form gravitational waves with strong enough effects to be barely detectable. Gravity waves have not yet been detected directly. However, the predicted influence of gravitational waves on a binary pulsar was measured by Hulse and Taylor (1975). A rapidly spinning neutron star emits two beams of radio waves along its magnetic axes. When the spin axis and magnetic axis are not identical, the radio beams are swept in two arches around the sky. If a beam path occasionally sweeps towards the Earth, a radio pulse can be detected, in this case with a period between pulses of 0.05903 sec. Thus, the star which is composed exclusively of neutrons is called a pulsar. This pulse period would be extremely stable except the observed period actually varies by several tens of microseconds as result of a Doppler shift indicating that the neutron star orbits a binary partner. Taylor and Hulse found that the orbit period is declining by about 75 millionths of a second per year (Hulse and Taylor 1975). These two stars are orbiting each other in a gradually smaller (therefore faster) orbit. The explanation is that these two massive stars are strongly accelerated by their circular orbits and thus required to lose energy in the form of gravity waves as predicted by general relativity. Peng has shown that the linearized general relativity equations produce equations similar to Maxwell's equations (Peng 1990). This will give rise to gravitational waves emission. It is believed that current technology now enables the direct detection of gravity waves. Gravitational waves are ripples in the fabric of space and time produced by violent events in the distant universe, such as the collision of two black holes or shockwaves from the cores of supernova explosions. Such gravity waves could bring with them information about their cataclysmic origins, as well as invaluable clues as to the nature of gravity.

We have recently found an analogy between electromagnetism and gravitation. According to this analogy electromagnetic phenomena are governed by analogous formula as gravitational phenomena (Arbab 2009a). This analogy is manifested in the formulae existing between the two paradigms. Gravitation is like electromagnetism, both are long range interactions. Coulomb and Newton law of gravitation are similar. However they are dissimilar since gravity attracts always while electricity can attract or repel. Accord-

A.I. Arbab (✉)
Department of Physics, Faculty of Science,
University of Khartoum, P.O. Box 321, Khartoum 11115, Sudan
e-mail: aiarbab@uofk.edu

A.I. Arbab
Department of Physics and Applied Mathematics,
Faculty of Applied Sciences and Computer,
Omdurman Ahlia University, P.O. Box 786, Omdurman, Sudan

ing to Einstein's theory of gravitation, light is deflected inward when intercepted by a gravitating objects, while alpha particles deflected outward when passing by a nucleus. The two formulae governing these phenomena are shown to be equivalent in the electrogravity analogy (Arbab 2009a; Arbab 2004c). At the present time there exists a quantum theory of electrodynamics, but no quantum theory of gravitation. Scientists tried to linearize Einstein's theory of gravitation and compare it with Maxwell's equation. It is assumed that both theories are governed by similar set of equations (Peng 1990). This method produces a system of equations implying a negative energy density of the gravitating system. This is rather a bizarre feature for gravitation. In our recent model, we however did not encounter such problems (Arbab 2009a). We have found that the unification of gravity with electromagnetism requires a prior unification with hydrodynamics (Arbab 2009b). Consequently, the three phenomena are self-similar, i.e., the mathematical formulae describing one phenomenon will imply its applicability to the other phenomenon. We aim in this work to apply this analogy to explore gravitational radiation emitted by massive objects and compared it with the power calculated from the general theory of relativity.

2 Gravitational radiation

The gravitational wave is a fluctuation in the curvature of space-time which propagates as a wave, traveling outward from a moving object or system of objects. Gravitational radiation is the energy transported by these waves. Important examples of systems which emit gravitational waves are binary star systems, where the two stars in the binary are white dwarfs, neutron stars, or black holes. Although gravitational radiation has not yet been directly detected, it has been indirectly shown to exist. This was the basis for the 1993 Nobel Prize in Physics, awarded for measurements of the Hulse-Taylor binary system (Hulse and Taylor 1975). The gravitomagnetic field is governed by the equations (Arbab 2009a)

$$\vec{\nabla}\cdot\vec{E}_g=\frac{\rho_m}{\varepsilon_g},\qquad \vec{\nabla}\cdot\vec{B}_g=0, \tag{1}$$

and

$$\vec{\nabla}\times\vec{B}_g=\mu_g\vec{J}+\frac{1}{c^2}\frac{\partial\vec{E}_g}{\partial t},\qquad \vec{\nabla}\times\vec{E}_g=-\frac{\partial\vec{B}_g}{\partial t}, \tag{2}$$

where $\varepsilon_g=\frac{1}{4\pi G}$ and $\mu_g=\frac{1}{\varepsilon_g c^2}$. The gravitomagnetic waves carries energy and momentum densities given by

$$u=\frac{1}{2}\varepsilon_g E_g^2+\frac{1}{2\mu_g}B_g^2,\qquad \vec{S}=\frac{\vec{E}_g\times\vec{B}_g}{\mu_g}. \tag{3}$$

The gravitomagnetic field $\vec{B}_g$ created by a gravitating object is given by (Arbab and Satti 2009)

$$\vec{B}_g=\frac{\vec{v}\times\vec{E}_g}{c^2}. \tag{4}$$

In electromagnetism, when a spinning charged particle (a magnetic dipole) is placed in a magnetic field (B), the dipole will precess with Larmor frequency, $\omega=\frac{q}{2m}B$. Owing to the existing analogy between gravitation and electromagnetism, when a gravitating object (dipole) is placed in a gravitomagnetic field the dipole will precess with an analogous Larmor frequency given by $\omega_g=\frac{B_g}{2}$, where $q\Leftrightarrow m$ (Arbab 2009a). Notice here that unlike the magnetic field B, the gravitomagnetic field, B_g is measured in rad/sec. The power delivers by the electric dipole radiation has a corresponding radiation in gravitation. We call this radiation, the gravitational Larmor radiation. We believe that it is governed by the same rules.

We know that the electromagnetic waves are able to carry energy, momentum, and angular momentum. By carrying these away from a source, waves are able to rob that source of its energy, linear or angular momentum. Gravitational waves perform the same function. Thus, for example, a binary system loses angular momentum as the two orbiting objects spiral towards each other—the angular momentum is radiated away by gravitational waves.

According to Larmor theory an accelerating (a) charged particle emits an electromagnetic radiation with power (Bertschinger and Taylor 2006; Larmor 1897)

$$P_{\rm em}=\frac{2}{3}\frac{q^2a^2}{4\pi\varepsilon_0c^3}. \tag{5}$$

According to our analogy, $k=\frac{1}{4\pi\varepsilon_0}\to G$, $q\to m$, one has a power generated by an accelerating (spinning) mass as (Arbab 2009a)

$$P_{hs}=\frac{2}{3}\frac{Gm^2a^2}{c^3}. \tag{6}$$

This can be casted in the form

$$P_{hs}=\frac{2}{3}\frac{Gm^2}{c^3}\omega^4R^2, \tag{7}$$

where $E_g=a=\omega^2R$, R is the radius of the body and ω is its angular velocity.

Equation (7) can be written as

$$P_{hs}=\frac{2}{3}\frac{Gm^2c}{R^2}\left(\frac{v}{c}\right)^4=\frac{2}{3}\,c\,F_g\left(\frac{v}{c}\right)^4, \tag{8}$$

where $v=\omega R$ is the velocity of the spinning mass at the surface, and $F_g=\frac{Gm^2}{R^2}$ is the self-gravitational force holding the spinning mass. This in fact represents the relativistic correction to the Newtonian power. This relation agrees with the luminosity of a galaxy found by the Tully Fisher law (Tully and Fisher 1977).

The power radiated by the Earth due to its spin, where $R = 6378$ km, $\omega = 7.27 \times 10^{-5}$ rad s^{-1}, is $P_{hs} = 6.7 \times 10^{10}$ W. This energy can be compared with the dissipation energy of the Earth due to its despinning, because of tidal forces raised by the Moon which is 3.0×10^{12} W. For the Sun one finds, $P_{hs} = 2.26 \times 10^{20}$ W, while the Sun luminosity is 3.8×10^{26} W. It can be compared with Jupiter which generates a power of $P_{hs} = 2.5 \times 10^{19}$ W.

It is an amazing coincidence that the power radiated by the Universe during its initial expansion (Planckian period) and at the present time is the same and is equal to $P_{hs} = 10^{52}$ W. That is so because the Planckian acceleration and the present accelerations are respectively, $a_0 = 10^{-10}$ m s^{-2} and $a_{\rm Pl} = 10^{51}$ m s^{-2} Arbab (2004, 2005). Notice however that $a = \omega^2 R = H^2 R$, where $H = 10^{-18}$ rad s^{-1} is the Hubble constant and $R = 10^{26}$ m is the universe radius. This present acceleration can be obtained from the relation $a = \frac{Gm}{R^2} = 10^{-10}$ m s^{-2}, where $m = 10^{53}$ kg is the universe mass. We remark that this coincidence is embedded in that fact that the maximal power is attained by the universe only. This implies that the force holding the universe at Planck time is the same as the one holding it now. The value of this force is 10^{43} N.

The centripetal acceleration of an orbiting body of mass m about a massive body of mass M is given

$$a = \frac{GM}{r^2} \tag{9}$$

so that the gravitational power radiated by the orbiting object is given by

$$P_{ho} = \frac{2}{3}\frac{G^3}{c^3}\frac{m^2M^2}{r^4}, \qquad P_{ho} = \frac{2}{3}\frac{G}{c^3}F_g^2,$$
$$F_g = \frac{GmM}{r^2}. \tag{10}$$

In terms of the orbital velocity, (12) yields

$$P_{ho} = \frac{2}{3G}\left(\frac{m}{M}\right)^2\left(\frac{v}{c}\right)^3 v^5. \tag{11}$$

Accordingly, the gravitational orbital power radiated by the Earth–Sun system is equal to 2.05×10^9 W. The power delivers by the Sun, owing to (8), is 1.8×10^{17} W. This can be compared with the gravitational power radiated by the Earth due to spin which is 6.7×10^{10} W. The orbital gravitational power radiated by Jupiter, owing to (11), is 1.14×10^{12} W.

For the binary pulsar PSR 1913 + 16 which is a system of two neutron stars having an orbital period of 7.75 hours at a distance of 1.95×10^9 m, one has a power of 4.1×10^{28} W, which is the typical value for x-ray luminosity of an x-ray pulsar (Hulse and Taylor 1975). This huge power is radiated away (lost) in a form of a gravitational radiation. Consequently, the orbital distance and the period of the system will decay with time.

Consider two masses m_1 and m_2, and they are separated by a distance r. According to the general theory of relativity, the power radiated off by this system is given by Bertschinger and Taylor (2006)

$$P = \frac{dE}{dt} = -\frac{32}{5}\frac{G^4}{c^5}\frac{(m_1m_2)^2(m_1+m_2)}{r^5}. \tag{12}$$

Due to this energy loss the orbital distance will decay by (Arbab 2009b)

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{(m_1m_2)(m_1+m_2)}{r^3}. \tag{13}$$

According to the above formula, the gravitational energy radiated by the Earth-Sun system is about 313 W.

The maximal power delivered by a gravitating object is given by (Arbab 2004; Arbab 2005)

$$P_{\max} = \frac{c^5}{G}. \tag{14}$$

If spinning gravitational objects emits the gravitational energy with the maximum power, then (7) and (9) yield

$$\omega_{\max} = \sqrt[4]{\frac{3}{2}}\frac{c^2}{\sqrt{GmR}} \tag{15}$$

so that the minimum wavelength of the emitted energy is

$$\lambda_{\min} = \sqrt[4]{\frac{2}{3}}\frac{2\pi}{c}\sqrt{G\,m\,R}. \tag{16}$$

Using (7), that the intensity of the radiated energy is given by

$$I_h = \frac{Gm^2}{6\pi\,c^3}\omega^4. \tag{17}$$

This formula resembles the Rayleigh-Jeans law for the black-body radiation at low frequency. For this reason we expect that (17) might not give the correct value for fast spinning objects (e.g., pulsars). Planck formula may be the appropriate intensity distribution of the gravitational wave radiated by all spinning masses. Equation (17) can be compared with Stefan law, where $\omega \propto \frac{1}{\lambda} \propto T$, or

$$T = \frac{a}{2\pi c}\omega, \quad a = 2.898 \times 10^{-3} \text{ m K}, \tag{18}$$

which is the Wien-displacement law. If we assume that this intensity is radiated like a black body at a temperature T (in kelvin), then one has

$$I_h = \sigma_g T^4, \quad \sigma_g = \frac{8\pi^3 cGm^2}{3a^4} = 2.345 \times 10^{10} \text{ m}^2, \tag{19}$$

where σ_g is the 'gravitational' Stefan constant. It is a characteristic constant of each gravitating body. Now if we equate the intensity in (17) with electromagnetic intensity and plug the values of the constants, one arrives at

$$T = 3.89 \times 10^{-8}\sqrt{m}\omega. \tag{20}$$

Accordingly, the Earth will radiate like a black-body having at a temperature of 6.9 K and Jupiter at 296 K.

Inserting the numerical values of the physical constants, one obtains

$$I_h = 5.24 \times 10^{23}\mu^2\omega^4, \tag{21}$$

where $\mu = \frac{m}{M_\odot}$. According to the above formula the gravitational intensity (energy flux) radiated by the Sun due to its spin is 37.0 $\mathrm{W\,m^{-2}}$ and by the Earth is 2.47 $\mu\mathrm{W\,m^{-2}}$. However, Jupiter radiates with 363 $\mathrm{W\,m^{-2}}$, which is about 10 times that of the Sun. Jupiter receives an energy from the Sun which is about 55 $\mathrm{W\,m^{-2}}$. Hence, Jupiter radiates more energy than it receives. Note that the intensity of solar radiation received on the Earth's surface is 1366 $\mathrm{W\,m^{-2}}$. The universal energy flux at the present time is $I_0 \sim 1$ $\mathrm{mW\,m^{-2}}$. The ratio of the intensity of the gravitational energy radiated during Planck time to the one radiated during the present time is so huge. This yields the value of 10^{122}. This ratio coincides with the ratio of vacuum energy at Planck time and the present time that particle physicists have found. It is not clear here what source of radiation this energy is radiated, but most probably in gravitational waves!

3 Concluding remarks

We have used the analogy between electromagnetism and gravity to arrive at the power radiated by spinning and orbital gravitational objects. Gravitating objects with mass m and radius R emit a gravitational wave with a minimum wavelength, $\lambda \propto \sqrt{mR}$.

References

Arbab, A.I.: A quantum universe and the solution to the cosmological problems. Gen. Relativ. Gravit. **36**, 3565 (2004)

Arbab, A.I.: A cosmic quantum mechanics. Afr. J. Math. Phys. **2**, 1 (2005)

Arbab, A.I.: A unified model of gravity and electromagnetism, unpublished (2009a)

Arbab, A.I.: On the analogy between the electrodynamics and hydrodynamics using quaternions. In: The 14th International Conference on Modelling Fluid Flow (CMFF'09), Budapest, Hungary, 9–12 September, 2009b (to appear)

Arbab, A.I.: A phenomenological model for the precession of planets and deflection of light, unpublished (2009c)

Arbab, A.I, Satti, Z.A.: On the generalized Maxwell equations and their prediction of electroscalarwave. Prog. Phys. **2**, 8 (2009)

Bertschinger, E., Taylor, E.F.: http://www.eftaylor.com/grsoftware/GravWaves17July06.pdf (2006)

Hulse, R.A., Taylor, J.H.: Astrophys. J. **195**, L51 (1975)

Larmor, J.: On a dynamical theory of the electric and luminiferous medium. Philos. Trans. R. Soc. **190**, 205 (1897)

Peng, H.: Gen. Relativ. Gravit. **22**, 609 (1990)

Tully, R.B., Fisher, J.R.: Astron. Astrophys. **54**, 661 (1977)