ABSTRACT

Under octahedral stresses and strain increment vectors, a soil possessing both cohesive and frictional properties is considered to be sliding instantaneously under drained conditions along the resultant strain increment vector direction at an arbitrary dilation angle $\psi$, to the vertical. Cohesive properties are represented by a strength in pure tension.

Considerations of the sliding mechanism allows the derivation of a flow rule relating stresses and strain increments. It is also shown that the angle between the resultant stress vector and the normal to the sliding surface is $\phi_{CV}$, the equivalent of the critical state angle $\phi_{CS}$, as obtained in the t-z plane.

Using the mathematical theory of plasticity in conjunction with the derived flow rule, a plastic potential function is obtained by the direct integration of the developed partial differential equation. Since along the sliding surface only shear stresses exist, the yield function is considered to be a function of the shear stress.

A reduction factor—which is to be assumed a function of the normal stress—is applied to the sliding surface shear-stress and taken to create the radius of the yield locus. The yield surface created travels with the current yield point and is changing all the time and as such is a kinematic one. Both the plastic potential and the yield functions satisfy the normality conditions at the current yield point resulting in an associated flow rule.

The hardening parameter is obtained from considerations of the void ratio—normal stress responses of the material. A complete stress strain increment relation is obtained which depends only on the material properties $\phi_{CV}, C_C, C_s$, the straining conditions, the initial stress levels and the applied stress increments.

The shape of the yield surface when intersecting the $\Pi$ plane, may be obtained for any material of known $\phi_{CV}$, by using the square root of the yield function and making comparisons at equal dilation angles and normal stress values but at varying Lode's parameter of stress $\mu$, values. The surface thus obtained resembles the experimental surface. This yield surface is assumed to be unique and independent of the inclination $\alpha$ of the major principal stress $\sigma_1$ to the vertical. The effect of the inclination would only be to cause the current yield point to rotate and travel along the yield surface. This leads to a corrected $\theta$ and hence $\mu$ values to be used instead of the original ones satisfying the condition ($\theta$ corrected = $\theta + \alpha$) effecting the change of strength as noted by experiment.

Predicted maximum angles of shearing resistance for Han River Sand ($\phi_{CV} = 32.8^\circ$) at a variety of initial porosities and confining pressures as well as at different $\alpha$ values, compare well with triaxial compression, triaxial extension and hollow cylinder experimental results. The maximum variation being close to $\pm2^\circ$. A better accuracy may be obtained if a more appropriate yield function reduction factor is found.

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NOMENCLATURE

\( \sigma_1, \sigma_2, \sigma_3 \) total major, intermediate and minor principal stresses

\( \sigma'_{1,2} \) effective major, intermediate and minor principal stresses

\( T \) octahedral shear stress

\( \frac{1}{3} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{\frac{1}{2}} 

\( P \) total normal stress

\( \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \)

\( P' \) effective value of \( P \)

\( P_0, P'_0 \) initial values of \( P \) and \( P' \)

\( \Delta \ldots \) incremental value of......

\( t = \frac{1}{2} (\sigma_1 - \sigma_3) \)

\( s' = \frac{1}{2} (\sigma_1 + \sigma_3) \)

\( R = \sigma_1' / \sigma_3' \)

\( \Pi \) plane octahedral shear stress plane at constant \( P \) value

\( b = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3) \)

\( \hat{\theta} \) inclination of \( T \) in the \( \Pi \) plane

\( g = 1 - 2b = -\sqrt{3} \tan \theta = \frac{(2 \sigma_2 - \sigma_1 - \sigma_3)}{(\sigma_3 - \sigma_1)} \)

\( \varepsilon'_1, \varepsilon'_2, \varepsilon'_3 \) major, intermediate and minor principal strain and increments

\( \nu = \varepsilon'_1 + \varepsilon'_2 + \varepsilon'_3 = \) volumetric strain increment

\( \varepsilon = \frac{1}{3} \nu \)

\( \Omega = \frac{(\varepsilon'_1 + \varepsilon'_3)}{2} \)

\( \zeta = \frac{(\varepsilon'_1 - \varepsilon'_3)}{2} \)

\( \nu = \frac{(2 \varepsilon'_2 - \varepsilon'_1 - \varepsilon'_3)}{(\varepsilon'_3 - \varepsilon'_1)} \)

\( g \) plastic potential function

\( f \) yield function

\( F \) square root of \( f \)

\( h, \lambda \) hardening parameters

\( H_g, H_f \) hardening constants respectively for \( g \) and \( f \)

\( \phi, \mu \) basic angle of interparticle friction

\( \phi_{cr} \) critical state angle of friction

\( \phi_{max} \) maximum angle of friction

\( \alpha \) inclination of the major principal stress to the vertical

\( e \) current void ratio

\( C_c \) coefficient of consolidation

\( C_s \) coefficient of swelling

INTRODUCTION

It has long been established that the mathematical theory of plasticity rules could be applied to soils with success. The critical state theory is a very good example of plasticity theory applications for solving problems related to clays. It has been shown both theoretically and experimentally that for soils, including granular materials, a flow rule exists; and so do plastic potential and yield surfaces. Hardening parameters were also found that relate to actual soil behavior. In addition the assumption of coincidence of the principal stress direction to those of the strain increments was shown to be acceptable. The works of Kirkpatrick (1957), Rowe (1962), Horne (1965), De Jesselin De Jong (1976), Hubelac (1966), Roscoe, Basset and Cole (1967) are but a few of the examples to be quoted in this...
of plasticity theories to soils may be found in Ismail (1997). In the following, a new plasticity theory using octahedral stresses and strain increments is derived for soils.

**SIGN CONVENTION**

Compressive stresses and strains are positive, clockwise shear stresses and anticlockwise shear strains are positive.

**DERIVATION OF THE FLOW RULE FOR DRAINED CONDITIONS**

Figure 1 shows a stress point, A, in a material possessing both frictional and cohesive properties, acted upon by a shear stress $T$ and a normal stress $P$. The resultant applied stress vector is $OA$. To take care of the tensile stress, resulted from the cohesive properties of the material, the origin of coordinates is transferred to $O'$ where $OO'$ is equal to $H$. The resultant stress vector at $A$ would then be $O'A$. If the stress system causes compressive strains and a resultant strain increment vector $AC$, acting at such an angle as $\Psi$, to the vertical, then the instantaneous sliding plane will be along $AC$. The resultant stress $O'A$ may be resolved into two components $AE$ along the direction $AC$ and $OE$ at right angles to it. The shear stress $AE$ may be referred to as $T_{ce}$, and the stress $OE'$, normal to the instantaneous sliding plane, as $P_{ce}$. Since there is no component of motion along the normal to the plane of sliding it becomes obvious that the angle $AOE'$ will be $\phi_{ce}$, the equivalent of the critical state, $\phi_{cv}$ of the t-s plane.

Resolution of the stresses gives:

$$P_{ce} = (P + H) \cos \Psi - T \sin \Psi$$  \hspace{1cm} (2a)

From the figure, the ratio:

$$\frac{R}{\rho + H} = \tan (\phi_{ce} - \Psi)$$

Denoting $T/(P + H)$ by $r$, then:

$$r = \frac{\tan \phi_{ce} - \tan \Psi}{1 + \tan \phi_{ce} \tan \Psi}$$  \hspace{1cm} (3a)

and

$$\frac{\delta}{\theta} = \tan \Psi$$  \hspace{1cm} (4a)

From (3a) and (4a) and denoting $\tan \phi_{cv}$ by $\delta$

$$\tan \Psi = \frac{\delta - \delta}{1 + \delta}$$ \hspace{1cm} (5a)

If on the other hand the applied stress system causes a dilating strain increment in figure 2, then:

$$T_{ce} = T \cos \Psi - (P + H) \sin \Psi$$ \hspace{1cm} (1b)

$$P_{ce} = (P + H) \cos \Psi + T \sin \Psi$$  \hspace{1cm} (2b)

$$r = \frac{\tan \phi_{ce} + \tan \Psi}{1 + \tan \phi_{ce} \tan \Psi}$$  \hspace{1cm} (3b)

$$\frac{\delta}{\theta} = \tan \Psi$$  \hspace{1cm} (4b)

Leading to:

$$-\tan \Psi = \frac{\delta - \delta}{1 + \delta}$$ \hspace{1cm} (5b)

Equations 5 may be rewritten as:
\[
\tan \varphi = \frac{1}{\theta} \frac{\delta - \tau}{1 + \delta x} \quad (5)
\]

This equation relates the current stresses with the resulting strain increment ratio and as such is the flow rule of the material under consideration.

THE PLASTIC POTENTIAL FUNCTION

The mathematical theory of work hardening plasticity relating the chosen stresses and strain increments may be written as:

\[
t = h - \frac{\partial g}{\partial (\delta + \Delta \delta)} \cdot df = \lambda + \frac{\partial g}{\partial (\delta + \Delta \delta)} \cdot \Delta \tau \quad (6)
\]

\[
\theta = h - \frac{\partial g}{\partial \tau} \cdot df = \lambda + \frac{\partial g}{\partial \tau} \cdot \Delta \tau \quad (7)
\]

where \(g\) is the plastic potential and \(f\) is the yield function.

\[
\text{and} \quad df = \frac{\partial \sigma}{\partial (\delta + \Delta \delta)} \cdot \Delta \delta, \frac{\partial \sigma}{\partial \tau} \cdot \Delta \tau
\]

\[
\text{and} \quad \lambda = h, df
\]

dividing 6 by 7:

\[
\frac{t}{\theta} = \frac{\partial g}{\partial (\delta + \Delta \delta)} / \frac{\partial g}{\partial \tau} \quad (9)
\]

equating 5 and 9:

\[
\frac{\partial g}{\partial \tau} / \frac{\partial g}{\partial \tau} = \frac{\partial g}{\partial \tau} / (\delta + \Delta \delta) \quad (10)
\]

but:

\[
\frac{\partial g}{\partial \tau} = \frac{\partial g}{\partial \tau} \cdot \frac{\delta - \tau}{1 + \delta x} \cdot \frac{\delta - \tau}{\partial (\delta + \Delta \delta)} \cdot \Delta \tau
\]

Hence

\[
\frac{(\delta - \tau)}{(1 + \delta x)} \cdot \frac{\partial g}{\partial \tau} = (\delta + \Delta \delta) \cdot \frac{\partial g}{\partial (\delta + \Delta \delta)} \quad (11)
\]

Solving the differential equation 11, we obtain:

\[
g = g(\beta + \Delta \beta)^{\alpha} \cdot (1 + \delta x) \cdot \exp \left(\frac{\delta}{\Delta \tau} \right) \quad (12)
\]

[\(a (\tan^{-1} \xi) / \delta\) - \(H_2 = 0\)]

For an even function \(a\) may be taken as 2, hence:

\[
g = (\beta + \Delta \beta)^{2} \cdot (1 + \delta x) \cdot \exp \left(\frac{2 \tan^{-1} \xi}{\delta} \right) \quad (13)
\]

Differentiation gives:

\[
\frac{\partial g}{\partial (\delta + \Delta \delta)} = \frac{2}{\delta} (\beta + \Delta \beta) \cdot (1 + \delta x) \cdot \exp \left(\frac{2 \tan^{-1} \xi}{\delta} \right) \quad (14)
\]

\[
\frac{\partial g}{\partial \tau} = \frac{2}{\delta} (\beta + \Delta \beta) \cdot (1 + \delta x) \cdot \exp \left(\frac{2 \tan^{-1} \xi}{\delta} \right) \quad (15)
\]

dividing 14 by 15 we obtain the flow rule of equations 10 and 5 relating the value of the outside normal to the plastic potential to the current stress point. The resulting plastic potential curves as obtained from equation 13, are shown in Figure 3.

THE YIELD FUNCTION

The stress, \(T_{cve}\) acting on the sliding plane satisfies normality and a circle with radius \(T_{cve}\) and center \(E\) (Fig.4) may be considered as the yield locus, \(F\), where:

\[
F = T_{cve} - H = F_{cve} - H = 0 = \left(\tau \cos \varphi + \sigma \sin \varphi\right)^2 - H_1 \quad (16a)
\]

and since \(T_{cve} = \delta P_{cve}\) then:

\[
\Psi = 0, \quad H_2 = (\delta P_{cve})^2 \quad (16b)
\]

Treating \(\Psi\) as an instantaneous constant
Differentiating (16b) 

\[ \frac{\partial \sigma}{\partial (p+H)} = \frac{\partial \epsilon}{\partial (p+H)} \tan \psi + \frac{\epsilon}{\sigma} \]  

leading to the flow rule as before. However, the magnitude of the hardening constant, \( H \), as obtained from equation 16b is not exactly compatible with that of the plastic potential function, \( H_p \), in magnitude as obtained from equation 13 and reduction factors need to be introduced. Consider the reduction factors \( a \) and \( b \) and the yield function:

\[ f = (T \cos \psi + a (p+H) \sin \psi)^2 - H_p \epsilon \]  

(20)

Taking \( H _p \) as a reduced form of the previous one, the radius of the new yield locus (Fig. 5):

the yield function, \( f \), may be written as:

\[ f = (T \cos \psi + a (p+H) \sin \psi)^2 - 
[b + \delta (p+H) \sin \psi]^2 \]  

(21)

Again:

\[ \frac{\partial f}{\partial (p+H)} = 2 (T \cos \psi + a (p+H) \sin \psi) \cdot a \cdot \sin \psi \]  

(22)

\[ \frac{\partial f}{\partial \psi} = 2 (T \cos \psi + a (p+H) \sin \psi) \cdot \cos \psi \]  

(23)

Dividing:

\[ \frac{\partial \sigma}{\partial (p+H)} = \frac{\partial \epsilon}{\partial \psi} \tan \psi + \frac{\epsilon}{\sigma} \]  

a reduced form of \( \dot{\epsilon} / \dot{\theta} \) but again satisfying normality.

**THE PEAK STRESS RATIO**

The peak stress ratio may be obtained

where \( \delta / \theta \) is the maximum strain increment ratio and \( a \) and \( b \) are reduction constants relating to the material considered and may depend on the degree of anisotropy, the straining conditions and the orientation of the principal stress direction to the vertical. Dependence on the porosity is already taken care of through the maximum applied strain increment ratio. The value of \( b \) may be found from the equation:

\[ \delta / \theta = \text{constant} \]  

(21a)

Hence \( b = 1 \)

The Value of \( r \) and \( a \):

\[ r = \sqrt{(6 + 2\mu) / ((3 - \mu) R + 3\mu)} \]

but at \( \phi \) conditions, \( R = K_{cv} \). Then:

\[ \delta = \tan \phi, r = \sqrt{(6 + 2\mu) / [(3 - \mu) K_{cv} + 3\mu]} \]  

(26)

where \( K_{cv} = \tan^2 (\frac{\mu}{4} \cdot \frac{\phi}{2}) \)  

(27)

The value of the reduction constant \( a \) may be found by trial and error after comparing a few experimental results with the theoretical predictions obtained using equations 25a using 26 and some chosen
for $a = (1 + \delta^2)/[K_{\mu}(1 + 5.75 \sin a)]$. (28)

trials gave

$$a = 1 \quad \ldots \ldots$$ (29)

$K_{\mu} = \sqrt{(6 + 2 \mu^2),(K_{\mu} - 1)}$ / 

$$[3 - \mu, K_{\mu} + 3 + \mu]$$ (30)

where $K_{\mu}$ = value of $R$ at ($\phi = \phi_0$)

$$K_{\mu} = \tan^2(\frac{\pi}{4}, \frac{\phi_0}{2})$$ (31)

$$\mu = -\sqrt{(3)} \tan^2\theta_{\text{corrected}}$$ (32)

which requires a knowledge of $\theta$ before results could be obtained.

where

$$\theta_{\text{corrected}} = \theta + \alpha$$ (33)

$t = s'$ plane presentation:

Using the $s'$ - $t$, $\Omega$, $\zeta$ terminology, noting that $b = 1$, equation 25(a) may be rewritten as:

$$t = \frac{\left(4 \sin(\mu + \varphi) \sqrt{(\delta(1 + \delta^2), (1 + x^2) / \sqrt{(6 + 2 \varphi^2) - a(3 \frac{\Omega}{\zeta} - \varphi)}}ight)}{\mu[\delta(1 + \delta^2), \sqrt{(6 + 2 \varphi^2) - a(3 \frac{\Omega}{\zeta} - \varphi)}]}$$ (25b)

which requires a knowledge of $\theta$ before results could be obtained.

Under hydrostatic compression conditions:

$$r = \Delta T = 0$$

and equation 39 reduces to:

$$\theta = \frac{3}{3} \frac{d(\Delta V)}{d \varphi}$$ (42)

$$= \frac{(C_1 - C_2)}{3(1 + \sigma)} \Delta \left(\frac{1}{(1 + \sigma)} \cdot \frac{1}{(1 + \delta^2)} \right)$$ (43)

In order to obtain the complete stress strain increment relationship as given by equations 6 and 7, the value of the hardening parameter, $h$, is required. Rewriting 6 and 7 using 14, 15, 22 and 23:

$$\Delta \left(\frac{1}{2} \Delta (p + \mu) \cdot (\delta - z) \cdot \frac{\delta}{\delta - z} \tan^2 \frac{\pi}{4}, \frac{\phi_0}{2}ight)$$ (34)

$$[\Delta \cos \varphi \Delta (p + \mu) \sin \varphi],$$

$$[\Delta \sin \varphi \Delta (p + \mu) + 2 \Delta \sin \varphi, \Delta T]$$

$$\theta = \frac{\delta}{\delta - z}$$ (35)

$$\sin \varphi = \frac{\delta - r}{\Delta T}$$ (36)

$$\sin \psi = \frac{(\delta - r)}{\sqrt{(1 + z^2)} \cdot \sqrt{(1 + \delta^2)}}$$ (37)

$$\cos \varphi = \frac{(1 + \delta)}{\sqrt{(1 + \delta^2)} \cdot \sqrt{(1 + \delta^2)}}$$ (38)

$$\varepsilon = \frac{4 \sin(\mu + \varphi) \sqrt{(\delta(1 + \delta^2), (1 + x^2) / \sqrt{(6 + 2 \varphi^2) - a(3 \frac{\Omega}{\zeta} - \varphi)}}}{\mu[\delta(1 + \delta^2), \sqrt{(6 + 2 \varphi^2) - a(3 \frac{\Omega}{\zeta} - \varphi)}]}$$ (39)

$$\theta = \frac{3 + \Delta \xi}{3} \cdot \xi$$ (40)

$$\theta = \frac{3 + \Delta \xi}{3} \cdot \xi$$ (40)

$\theta = \frac{\delta}{\delta - z} \tan^2 \frac{\pi}{4}, \frac{\phi_0}{2}$

$\mu[\delta(1 + \delta^2), \sqrt{(6 + 2 \varphi^2) - a(3 \frac{\Omega}{\zeta} - \varphi)}]$. (25b)
THE INTERSECTION OF THE YIELD SURFACE WITH THE (1) PLANE

To find the shape of the yield surface at the intersection with the (1) plane, equations 25 to 33 may be used to obtain the peak stress ratio, \( r \), for any one given applied strain increment. The value of \( \theta \) and \( k(\alpha) \) for any chosen value of Lode's parameter of stress \( \mu \) may be found respectively from equations 26 and 30 or figures 6 and 7. The resulting yield surfaces are shown in fig. 8. The resemblance to the reported experimental yield curves is evident.

COMPARISON BETWEEN THEORETICAL AND PREDICTED PEAK STRENGTH VALUES

Ham River Sand is chosen for comparison purposes. This is a granular material with the properties:

\[ \phi_{cv} = 32.8^\circ \]

Using equations 25 to 33 and any chosen peak strain increment value and different values of Lode's parameter of stress \( \mu \), the peak stress ratio, \( r \), may be calculated. The value of the peak major principal to minor principal stress ratio denoted by \( R \), may then be found from:

\[ R = \frac{3 - 2 \mu^2}{\mu^2 + (R - 1)} \]

where \( \theta = \tan^2 \left( \frac{\pi}{4} + \frac{\phi_{cv}}{2} \right) \)

from which the peak \( \phi_{\text{max}} \) may be obtained.

Theoretically calculated values of \( \phi_{\text{max}} \) for triaxial compression as compared to Green and Skemps's experimental data as reported by Bishop (1971) for a variety of confining pressures and initial densities can be seen from fig. 9. Fig 10 gives the comparison for the triaxial extension case compared to Reade's results reported by Reade (1971). Table 1 shows the results reported by Symes (1983) using the Imperial College illoow cylinder apparatus on the same sand at varying inclinations of the major principal stress direction to the vertical for four chosen different \( \mu \) values. It is seen that the theory predicts experimental results with a good degree of accuracy through the whole range of porosities considered and for a wide range of confining pressures and \( \alpha \) values. The maximum deviation being close to \( \pm 2^\circ \). It is thought that better accuracies are possible with a more appropriate choice of the reduction coefficient.
DEVELOPMENTS FOR UNDRAINED CONDITIONS

Since the proposed model is generated using a sliding mechanism under effective stress conditions, the same model may be sought to solve sliding under undrained conditions. However, since there is no volume change under undrained conditions, the volumetric strain increment vector, \( \dot{\varepsilon} \), may be replaced by a function related to the rate of change of pore pressure increment with respect to the applied stress increment.

CONCLUSIONS

1. The proposed model of a soil possessing both frictional and cohesive properties instantaneously sliding under drained conditions along the resultant strain increment vector direction at a dilation angle, \( \Psi \), does that under the equivalent of critical state conditions.

2. The geometry of sliding formed by the applied stresses and strain increments allows the derivation of a flow rule which is used in conjunction with the mathematical theory of plasticity to generate a plastic potential function by direct integration of the arising differential equation.

3. The plastic potential function obtained gives rise to convex curves, the outside normal of which automatically gives the correct strain increment vector.

4. A kinematic yield function has been obtained relating only to a reduced form of the shear stress acting along the plane of sliding.

5. The reduction factor is assumed to be a function of the degree of anisotropy, the straining conditions and the orientation of the major principal stress with respect to the vertical.

6. Using the derived flow rule, plastic potential and the proposed yield function, a complete stress strain increment mathematical relationship has been obtained in which the only unknown is the hardening parameter, \( \dot{\lambda} \).

7. By assuming that the obtained mathematical relationship applies even under hydrostatic compression conditions, the hardening parameter has been derived in terms of the material properties \( C_0, C_s, \phi_{sv} \) as well as the current void ratio.

8. The use of the complete stress strain increment relationship obtained requires a knowledge of:
   - the initial stress state
   - the applied increments of the octahedral shear and normal stresses, i.e. the stress path
   - the critical state angle (\( \phi_{cr} \))
   - the basic angle of internal friction (\( \phi_{\mu} \))
   - the coefficient of consolidation \( C_C \)
   - the coefficient of swelling \( C_S \)
   - the strength of the material in pure tension.
   - a chosen reduction factor, a

9. A first degree order of the yield function allows the prediction of the maximum attainable material strength in terms of \( (\phi_{max}) \) for any given density (i.e. maximum dilation angle, \( \Psi \)) and the chosen straining condition (i.e. \( \mu \) value). However, first the value of the reduction constant, \( a \), has to be evaluated from a few experimental results.

10. Using the first degree order of the yield
yield surface intersecting with the $\Pi$ plane obtained resembles experimental findings.

11 The yield surface intersection with $\Pi$ plane as obtained, seems to be unique. This is borne by its ability to predict the effects of principal stress rotations on the material strength as noted by experiment. The rotation of the major principal axes only causes the travel of the yield point along the yield surface with the effect that the Lode's parameter of stress changes to a new value obtained by using:

$$\theta_{\text{corrected}} = \theta + \phi.$$ 

12 The theoretically predicted maximum angles of shear strength for a variety of straining conditions for a wide range of porosities and confining pressures including principal stress rotations for Ham River Sand tested under triaxial compression, triaxial extension and hollow cylinder (including plane strain) show very good agreement with experiment. The maximum variation between theory and experiment is close to ($\pm 2\%$). Better accuracies may be obtained by using a more appropriate reduction factor.

ACKNOWLEDGEMENTS

Although the ideas and preliminary works on the development of the present theory started during the late seventies, serious developments were only made during the author's sabbatical leave at Imperial College, London, in 1987. The generous hospitality and facilities offered by professor J. B. Burland and the valuable discussions with him and his colleagues made it possible to arrive at solutions for most of problems tackled. The present final

PATENT

Approach is being made to patent the theory in the author's name.

REFERENCES

Table (1) Comparison between theoretical and experimental hollow cylinder results:

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Fig (1) S.I.R.E.S.S.E.S CAUSING COMPRESSION STRAINS

Fig (2) S.I.R.E.S.S.E.S CAUSING DILATION

Fig(3) PLASTIC POTENTIAL CURVES

Fig(4) YIELD LOCUS
Fig. (5) REDUCED YIELD LOCUS

Fig. (6) THE VALUE OF $K_{Me}$
Fig (7) The variation of $\delta$ with $\alpha$.

Fig (8) Yield surface intersection with the $\tau$ plane.
Fig (8) COMPARISON OF THEORETICAL & EXPERIMENTAL TRIAXIAL COMPRESSION RESULTS FOR HAM RIVER SAND

Fig (10) COMPARISON OF THEORETICAL & EXPERIMENTAL TRIAXIAL EXTENSION