
APPLICATION OF THE DIRECT DESIGN METHOD ON REINFORCED CONCRETE BEAMS SUBJECTED TO COMBINED TORSION BENDING AND SHEAR

Hussien Elarabi

Abstract: This research work is concerned with the problem of combined torsion, bending and shear. It consists of three linked parts. A sandwich approach is developed, in which all components of stress-resultants from different combinations of loadings on a reinforced concrete beam are lumped into four plates at its edges. This is to facilitate the application of the direct design method on beams subjected to combined bending, torsion and shear. The stress-resultants of these plates are computed by a three-dimensional finite elements computer program based on a 20-node isoparametric element developed in this study.

1. INTRODUCTION

Torsion, bending and shear forces can act simultaneously on a beam in any combination. During the last four decades the ability of concrete members to sustain torsional loads has become more appreciated, owing to the use of more advanced structures in which torsional moments of considerable magnitude arise. With the introduction of the ultimate strength design and refined with design, the need to take torsion into account emerged.

The rapid development of digital computers in the recent years put the finite element method in the position of a most acceptable powerful general technique and unified approach for the numerical solution of a very large variety of problems encountered in engineering.

In this study a three-dimensional finite element computer program is developed and applied to problems of torsional loading combined with bending and shear on reinforced concrete members.

The general procedure adopted by the codes of practice in design for torsion, bending and shear when acting simultaneously, is to treat each type of loading individually. This may results in overestimating the amount of reinforcement required for the structure.

In this research, a direct design method is proposed. The direct design method is a method used to compute the reinforcement required to resist the stress-resultants from certain combinations of loading acting on a structural element. To apply this concept to beams subjected to torsion, bending and shear, a sandwich approach is developed in which all components of stress-resultants are lumped into four plates at the edges of the beam. The stress vector is determined by the finite element program developed in this study. This method of design is compared with the specifications of the British Standard Code of Practice (BS8110-1985)(1) and the American Code of Practice (ACI 318-83)(2) by comparing their relative merits.

2. APPLICATION OF FINITE ELEMENT METHOD TO TORSION

2.1 General

Three types of forces can act on a beam, these are bending, shear and torsion. They can exist simultaneously in any combination as dictated by applied loads and geometry. Reinforcement in analysis, the greater use of novel structural forms by architects and the employment of ultimate strength rather than working stress design, had given explicit attention to torsion.

The torsion forces are three dimensional in nature. This complicates the situation for any non-linear analysis and requires careful attention.

As the major portion of the developed finite element programs was two dimensional, the application of the finite element method is very common for bending and shear. Very limited work has been reported on torsion, references (3,4) are the only two known to the author.

In this paper the three dimensional finite element method is applied for short term loading of reinforced concrete. The method is applied to a reinforced concrete beam under various load types including pure and combined torsion, bending and shear.

2.2 Program Description

The program (Master) is a finite element program written in Fortran 77 for the analysis of three dimensional non-linear structures using 20-node isoparametric brick elements for concrete and embedded bar elements for reinforcement.

All dimensions in subroutines were collected in one separate file containing all common blocks and included at the beginning of each subroutine.

(239)

A parameter controlling the identity of elements is considered so that if the data of the material properties for each element is identical, it need to be entered once.

A mesh generator for coordinates of corner and mid-side nodes as well as outside mesh numbering of nodes was implemented in the program.

Numerical integration for the element stiffness matrix is carried out using either 2x2x2, 3x3x3 or 4x4x4 Gauss quadrature rule.

The developed program permits application of two types of loads namely nodal point loads and distributed loads on element edges. The equilibrium equations were solved by the frontal techniques (5).

The principal stresses and strains and their corresponding directions were computed by subroutine STRESS3D.

A subroutine to compute the forces and moments on each surface of the element was added, subroutine SHELL which converts these forces and moments to stresses on the surface of the suggested plates designed and called within subroutine STRESS3D.

Subroutine INPLNED is added and called at the end of subroutine STRESS3D to compute the required reinforcement for certain structural elements using the direct design method discussed in the next section.

The input data needed to run the program for any type of problem is the control data plus the geometric data, the boundary conditions of the structure and the materials properties.

The output from the program consist of: displacements at nodal points as well as stresses, strains, principal stresses and principal strains at sampling points (Gauss points). Also it consists of forces and moments on each surface of each element, stresses on plates and the required reinforcement for the element.

3. THE DIRECT DESIGN METHOD

3.1 Introduction

The direct design method is an application of the plasticity theory for proportioning the reinforcement required to resist a combination of torsion, bending and shear on reinforced concrete members. The method depends on the stress-resultants only and is independent of the type of loading.

Clark's equations (6) used in this study depends on the in-plane forces acting on plates. A sandwich approach, in which all components of stress-resultants are lumped to four plates at the edges of a rectangular beam, is developed.

The direct design method is applied to cantilever beam subjected to different combination of torsion, bending and shear. The same beam with the same different types of loadings is designed according to the British Standard Structural use of Concrete (BSS110-1985) and the Building Code Requirements for reinforced concrete (ACI1218-83). The results of different types of design are then compared.

3.2 Clark's equations

Given the stress triad \((N_x, N_y, N_{xy})\) at any point in the plate, it is required to design reinforcement according to the lower bound theory of plasticity. A general case of providing reinforcement per unit length \(A_x\) and \(A_y\) in two directions \(x\) and \(y\) will be considered. Associated stress for these areas will be \(f_x\) and \(f_y\). With reference to Fig. (1) and Fig. (2), the following equations may be written:

\[
N_x = A_x f_x + A_y f_y \cos^2 \theta + \sigma_1 h \sin^2 \theta + \sigma_2 h \sin \theta \cos \theta
\]

\[
N_y = A_x f_x \sin^2 \alpha + \sigma_1 h \sin^2 \theta + \sigma_2 h \cos^2 \theta
\]

\[
N_{xy} = -A_y f_y \sin \alpha \cos \alpha + \sigma_1 h \sin \theta \cos \theta
\]

\[
+ \sigma_2 h \cos \theta \cos \theta
\]

where:

\(\sigma_1, \sigma_2\) are the concrete principal stresses and \(\alpha\) is always algebraically greater than \(\sigma_2\) and making angle \(\theta\) to the \(x\)-axis.

![Fig. (1): Sign convention for in-plane direct and shear forces per unit length](image)

![Fig. (2): Directions of reinforcement and principal concrete stresses](image)

On dividing through by the slab thickness \( h \) and defining the direction and shear stresses as:

\[
\sigma_x = \frac{N_x}{h}, \\
\sigma_y = \frac{N_y}{h}, \\
\tau_{xy} = \frac{N_{xy}}{h} 
\]  \hspace{1cm} (3.2)

and the reinforcement ratio as:

\[
\rho_x = \frac{A_x}{h}, \\
\rho_y = \frac{A_y}{h} 
\]  \hspace{1cm} (3.3)

we obtain:

\[
\sigma_x = \rho_x f_y + \sigma_c h \cos^2 \alpha + \sigma_t h \cos \theta \cos \phi + \sigma_s h \sin \theta \cos \phi \\
\sigma_y = \rho_y f_y + \sigma_c h \sin^2 \alpha + \sigma_t h \sin \theta \cos \phi + \sigma_s h \cos \phi \\
\tau_{xy} = -\rho_x f_y \cos \alpha - \sigma_c h \sin \theta \cos \phi + \sigma_s h \cos \theta \cos \phi 
\]  \hspace{1cm} (3.4)

Equations (3.4) contain seven unknowns. By predetermining some of these unknowns, these equations can be solved. The variables are predetermined according to the nature of the reinforcement: tensile, compressive, or no reinforcement. This yields nine possible combinations summarized in Table 1. It can be seen that a direct solution can be obtained for all cases except cases 1 and 4 where there are four unknowns to be obtained from three equations. An additional equation can be obtained by minimizing the total amount of reinforcement, i.e.

\[
\frac{\partial}{\partial \tan \theta} (\rho_x + \rho_y) = 0.0 
\]  \hspace{1cm} (3.5)

Since the tensile strength of concrete is assumed to be zero, the value of \( \sigma_c \) in Table 1 is taken as zero, when tension reinforcement is provided, and \( \sigma_c \) is taken as the value of the "yield" stress of concrete when compression reinforcement is provided so as to make optimum use of the concrete. Now the expression for area of reinforcement, principal concrete stresses, and \( \theta \) for each case can be obtained. Further details may be found in reference (6).

3.3 Application of the direct design method for beams

The cantilever beam shown in Fig. (3) is subjected to different load combination of torsion, shear, and bending. As the equations given by Clark (6) are applicable for plates only, the beam is divided into four plates at its boundaries, Fig. (4). The stress vector at the Gauss sampling points is obtained by the finite element program. A subroutine called FORCES is developed to compute the forces and moments on each surface of the element considered. This subroutine integrates the stresses at the sampling points over their corresponding surface using the 2x2 Gauss quadratic rule. The sign conventions for the forces and moments are shown in Fig. (5).

\[
f_w = 25 \text{ N/mm}^2 \\
f_y = 250 \text{ N/mm}^2 
\]

Fig. (3): Cantilever beam for the study of direct design method

These forces and moments are converted to stresses on the surface of the suggested plates using a subroutine called SHELL. The forces on the surfaces are lumped to stresses on the plates according to the following equations;

From figures 4, 5 and 6:

For plate 1, stresses on surface \( \alpha \) are:

Fig. (4): Division of the beam into plates

Table (1): Summary of various possible combinations of reinforcement (Ref. (6))

<table>
<thead>
<tr>
<th>Case</th>
<th>Reinforcement description</th>
<th>Known values</th>
<th>Method of solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Both tension</td>
<td>$f_x = f_y = f_z$, $\sigma_t = 0$</td>
<td>Minimization of $(\rho_x + \rho_y)$</td>
</tr>
<tr>
<td>2</td>
<td>No $x$ tension</td>
<td>$f_a = f_y$, $\rho_x = 0$</td>
<td>Direct solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_t = 0$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>No $x$ tension</td>
<td>$f_x = f_y$, $\rho_x = 0$</td>
<td>Direct solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_t = 0$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Both compression</td>
<td>$f_x = f_y = f_z$, $\sigma_t = f_e$</td>
<td>Minimization of $(\rho_x + \rho_y)$</td>
</tr>
<tr>
<td>5</td>
<td>No $x$ compression</td>
<td>$f_a = f_y$, $\rho_x = 0$</td>
<td>Direct solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = f_e$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>No $x$ compression</td>
<td>$f_x = f_y$, $\rho_x = 0$</td>
<td>Direct solution</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\sigma_x = f_e$</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$x$ tension $\alpha$ compression</td>
<td>$f_x = f_y$, $f_a = f_z$</td>
<td>Direct solution</td>
</tr>
<tr>
<td></td>
<td>$\alpha$ compression</td>
<td>$\sigma_x = 0$, $\sigma_t = f_e$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$x$ compression $\alpha$ tension</td>
<td>$f_x = f_y$, $f_a = f_z$</td>
<td>Direct solution</td>
</tr>
<tr>
<td></td>
<td>$\alpha$ tension</td>
<td>$\sigma_x = 0$, $\sigma_t = f_e$</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>No reinforcement</td>
<td>$\rho_x = \rho_y = 0$</td>
<td>Direct solution</td>
</tr>
</tbody>
</table>

Fig. (5): Sign convention for forces and moments

\[ \sigma_{yy} = \frac{F_y}{(2C_B)} - \frac{M_x}{(D_1 C_B)} \]
\[ \tau_{yx} = \frac{F_{yx}}{(2C_B)} - \frac{M_{yx}}{(D_1 C_B)} \]

(3.6)

Stresses on surface - \( yz \) - are:

\[ \sigma_{zz} = \frac{F_z}{(2C_L)} - \frac{M_y}{(D_1 C_L)} \]
\[ \tau_{zy} = \frac{F_{zy}}{(2C_L)} - \frac{M_{zy}}{(D_1 C_L)} \]

(3.7)

For plate (2), stresses on surface \( xz \) are:

\[ \sigma_{yy} = \frac{F_y}{(2C_B)} + \frac{M_x}{(D_1 C_B)} \]
\[ \tau_{yx} = \frac{F_{yx}}{(2C_B)} - \frac{M_{yx}}{(D_1 C_B)} \]

(3.8)

Stresses on surface - \( yz \) - are:

\[ \sigma_{zz} = \frac{F_z}{(2C_L)} - \frac{M_y}{(D_1 C_L)} \]
\[ \tau_{zy} = \frac{F_{zy}}{(2C_L)} - \frac{M_{zy}}{(D_1 C_L)} \]

(3.9)

For plate (1), stresses on surface \( xz \) are:

\[ \sigma_{yy} = \frac{M_x}{(B_1 C D)} \]
\[ \tau_{yx} = \frac{F_{yx}}{(2C_D)} - \frac{M_{yx}}{(B_1 C D)} \]

(3.10)

Stresses on surface - \( xy \) - are:

\[ \sigma_{zz} = \frac{M_y}{(B_1 C L)} \]
\[ \tau_{zy} = \frac{F_{zy}}{(2C_L)} - \frac{M_{zy}}{(B_1 C L)} \]

(3.11)

For plate (4), Fig. (6), stresses on surface \( xz \) are:

\[ \sigma_{yy} = -\frac{M_z}{(B_1 C D)} \]
\[ \tau_{yz} = \frac{F_{yz}}{(2C D)} + \frac{M_{yz}}{(B_1 C D)} \]

(3.12)

Stresses on surface - \( xy \) - are:

\[ \sigma_{zz} = \frac{M_y}{(B_1 C L)} \]
\[ \tau_{zy} = \frac{F_{zy}}{(2C L)} + \frac{M_{zy}}{(B_1 C L)} \]

(3.13)

Fig. (6): Stresses on plates

Where:
- \( C \): Thickness of the plate,
- \( L, B, D \): Dimensions of element, see Fig. (4),
- \( B_1, D_1 \): Lever arms of the moments measured from the centre of the plates, see Fig. (4),
- \( M_x, M_y, M_z \): Normal stresses on the surface of the plates,
- \( \tau_{xy}, \tau_{yx} \): Shear stresses on the surface of the plates,
- \( F_x, F_y, F_z \): Normal forces on the surface of the elements computed from the corresponding normal stresses,
- \( F_{xy}, F_{yx}, F_{xz} \): Shear forces on the surface of the elements computed from the corresponding shear stresses,
- \( M_x, M_y, M_z \): Moments around \( x, y \), and \( z \)-axis respectively,
- \( M_{xy}, M_{yx}, M_{xz} \): Twisting moments computed from the corresponding shear stresses.

Thus the beam problem is converted to a set of four plates under the action of membrane or in-plane stresses. The direct design method can now be applied.

3.4 Effect of the thickness of the shell on the distribution of reinforcement

It can be noted from the above that a basic parameter to be determined in the developed method is the plate thickness. An investigation was carried out to study its effect. A bending moment of 90 kNm is applied at the end of the 2.0 meters long cantilever of Fig. (3) and the corresponding reinforcement is obtained for a constant depth and varying thickness and secondly for a constant depth and varying thickness. In each case the thickness of the plate is varied from 1.0 mm to half the width (125 mm) with interval of 1.0 mm. This is plotted in Fig. (7) and Fig. (8) to show the relationship between the thickness and the amount of longitudinal reinforcement.

The above procedure was repeated for the case of pure torsion \( (T = 40 \text{ kNm}) \) and the case of combined torsion and bending moments \( (T = 40 \text{ kNm}, M = 90 \text{ kNm}) \). The results are shown in Fig. (9), (10), (11), and (12).

![Fig. (7): Relation between the plates thickness and the longitudinal reinforcement. Case of pure bending. (Width = 250 mm)](image)

![Fig. (8): Relation between the plate thickness and the longitudinal reinforcement. Case of pure bending. (Depth = 450 mm).](image)

Fig. (7) and Fig. (8) show that the value of the longitudinal reinforcement decreases as the thickness increases till it reaches a certain value after which it begins to increase with the increase of thickness. This value of minimum reinforcement represents the first point where no compressive reinforcement is needed.

Table (2) shows the thickness which gives the minimum reinforcement in the case of combined torsion and bending, pure bending and pure torsion. It can be seen from the table that the thickness for the combined case is greater than the thickness obtained from the pure cases and less than their sum.

<table>
<thead>
<tr>
<th>Load type</th>
<th>Value (kNm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment</td>
<td>90</td>
<td>40</td>
</tr>
<tr>
<td>Torque</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>Moment</td>
<td>90</td>
<td>50</td>
</tr>
<tr>
<td>Torque</td>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

The thickness which gives the minimum reinforcement is plotted against its corresponding depth at the time when the width is constant, as shown in Fig. (13). The best equation which simulates this relation can be put in the form:

\[ y = ax^2 \] (3.14)

Constant while the width was varied, (see Fig. (14) and Table (3).

Fig. (9) and Fig. (10) show the relationship between the thickness and longitudinal reinforcement in the case of pure torsion for varying depth and width respectively.

Fig. (11): Relation between the plate thickness and the longitudinal reinforcement. Case of combined bending and torsion, (width = 250 mm).

Fig. (12): Relation between the plate thickness and the longitudinal reinforcement. Case of combined bending and torsion (depth = 450).

By translation of this equation to a linear equation using logarithms, linear regression can be carried out to find the constants $a$ and $b$ for this particular problem. The results are shown in Table (3). Similar results and conclusions were obtained for the case of pure bending, when the depth was kept

It can be seen that the volume of longitudinal reinforcement increases as the thickness increases starting from a certain minimum thickness. The minimum thickness corresponds to the thickness for which the shear stress on the plate does not exceed 0.5 $f_c$. Fig. (13) and Fig. (14) show the relationship between the thickness which gives the minimum volume of reinforcement and the depth and the width respectively. These relations can also be simulated by equation (3.14). The constants $\alpha$ and $\beta$ for this particular case are given in Table 3. Fig. (11),(12),(13) and (14) show the results for the case of combined bending and torsion. They are in general similar to those obtained in the case of pure bending discussed above. The constants $\alpha$ and $\beta$ are given in Table 3.

It should be mentioned that the values of the constants $\alpha$ and $\beta$ are only applicable for the particular loading combinations discussed.

From the results it can be seen that the thickness of the plate is governed by two factors:
- the condition of minimum cover, and the
- practicality of the arrangement of reinforcement.

To satisfy the condition of minimum cover the plate thickness should be at least twice the cover although there is another minimum thickness which gives less reinforcement. Governed by the assumption of placing the reinforcement at the middle of the plate, the thickness is recommended to be twice the cover although a larger thickness gives less reinforcement.

![Diagram showing the relation between thickness and reinforcement](image)

**Fig. (13):** Relation between thickness which gives minimum longitudinal reinforcement and the corresponding depth.

**Fig. (14):** Relation between thickness which gives minimum longitudinal reinforcement and the corresponding width.

3.5 Verification of the developed direct design method

In this section the developed design software is tested. The same cantilever beam is used. The results obtained for the case of combined torsion and bending show that the values of shear stresses on plates (4) and (2) are simply the addition of that obtained in pure torsion to that obtained in pure bending, Table 4. It follows that the reinforcement is approximately the sum of the two individual pure cases.

Considering plate (3) it can be seen that the values of shear stresses for combined torsion and bending are equal to the algebraic sum of these values for each individual loading. The values of the percentage reinforcements in this plate is approximately equal to the subtraction of the percentage reinforcement for pure bending from that for pure torsion.

For plate (1) the reinforcement for combined torsion and bending is less than for pure torsion. This is because in this plate there is only a normal compressive stress obtained from bending added to a shear stress obtained from torsion. The compressive stress in this plane tends to reduce the effect of the shear stress. From the above discussion it can be concluded that the direct design method produces acceptable results.

3.6 Comparison between the direct design method and codes of practice

In this section a comparison of the results obtained by the direct design method with those
obtained by the British Standard Code of practice (BS8110-1985) and the American Code of practice (ACI319-83) for different moment/torque ratios is presented.

Seven cantilever beams with the same cross sectional area (450 × 250 mm) subjected to pure and combined torsion were designed according to the (BS 8110-1985), (ACI 319-83) and the direct design method. All safety factors in the two codes of practice were eliminated so as to have the same basis for comparison.

Table (5) shows the amount of longitudinal and transverse reinforcements for various moment/torque ratios. It also contains the percentage difference (DIF) between the longitudinal reinforcement computed by the direct design method (DMD) and the other Codes of Practice, and the percentage difference (DIF2) between the transverse reinforcement computed by the direct design method and the other Codes of Practice.

From the table it can be seen that for pure torsion the direct design method gives less reinforcement than the two codes of practice. BS8110 gives an amount of longitudinal and transverse reinforcements which are 1.47 and 1.83 times the respective value obtained by the direct design method.

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Value of $\alpha$</th>
<th>Value of $\beta$</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure torsion, $T = 40$ kNm</td>
<td>$12.257 \times 10^6$</td>
<td>-1.05</td>
<td>Constant width</td>
</tr>
<tr>
<td>Pure bending, $M = 90$ kNm</td>
<td>$19.629 \times 10^6$</td>
<td>-1.25</td>
<td>Constant depth</td>
</tr>
<tr>
<td>Combined torsion and bending, $M = 90$ kNm, $T = 40$ kNm</td>
<td>$65.669 \times 10^6$</td>
<td>-1.21</td>
<td>Constant width</td>
</tr>
<tr>
<td>Combined torsion and bending, $M = 90$ kNm, $T = 40$ kNm</td>
<td>$22.633 \times 10^6$</td>
<td>-1.15</td>
<td>Constant depth</td>
</tr>
<tr>
<td>Combined torsion and bending, $M = 90$ kNm, $T = 40$ kNm</td>
<td>$134.078 \times 10^6$</td>
<td>-1.31</td>
<td>Constant width</td>
</tr>
<tr>
<td>Combined torsion and bending, $M = 90$ kNm, $T = 40$ kNm</td>
<td>$15.667 \times 10^6$</td>
<td>-1.07</td>
<td>Constant depth</td>
</tr>
</tbody>
</table>

The corresponding ratios for the ACI are 1.62 and 1.42 respectively.

In the case of combined torsion and bending it can be seen that in general the difference in the longitudinal reinforcement between the direct design method and both codes of practice decreases as the moment/torque ratio increases.

The difference in transverse reinforcement obtained by ACI and BS8110 from that obtained by direct design method is high and almost constant for $MT$ ratios less than 6 and drops for value of $MT$ higher than 6.

From Table (5) it can be observed that the total volume of reinforcement in the beam obtained by the direct design is less than that obtained by the two codes of practice. Thus it can be stated that the direct design method requires less steel than the ACI and BS8110 codes of practice, bearing in mind the fact that no safety factors are involved in each case.

<table>
<thead>
<tr>
<th>Type of loading</th>
<th>Plate</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
<th>$\sigma_z$</th>
<th>$\sigma_x$</th>
<th>$\sigma_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure torsion, $T = 40$ kNm</td>
<td>1</td>
<td>0.0</td>
<td>-16.15</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td></td>
<td>2</td>
<td>0.0</td>
<td>-16.15</td>
<td>0.0</td>
<td>0.0</td>
<td>0.065</td>
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<td></td>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>-1.907</td>
<td>0.004</td>
<td>0.004</td>
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<td></td>
<td>4</td>
<td>0.0</td>
<td>-1.907</td>
<td>0.0</td>
<td>0.065</td>
<td>0.065</td>
</tr>
<tr>
<td>Pure bending, $M = 90$ kNm</td>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>-3.872</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>3.872</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>4.575</td>
<td>0.018</td>
<td>0.018</td>
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<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>-4.575</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>Combined torsion and bending, $M = 90$ kNm, $T = 40$ kNm</td>
<td>1</td>
<td>0.0</td>
<td>-16.52</td>
<td>-3.872</td>
<td>0.004</td>
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<td>2</td>
<td>0.0</td>
<td>16.52</td>
<td>3.872</td>
<td>0.015</td>
<td>0.062</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>3.599</td>
<td>0.014</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.0</td>
<td>0.0</td>
<td>-5.551</td>
<td>0.022</td>
<td>0.022</td>
</tr>
</tbody>
</table>

### Table 5: Reinforcement obtained by BS8110, ACI and DMD

<table>
<thead>
<tr>
<th>M (kNm)</th>
<th>T (kNm)</th>
<th>BS8110</th>
<th>ACI</th>
<th>DMD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Long Rein.</td>
<td>Trans Rein.</td>
<td>DIF1 %</td>
</tr>
<tr>
<td>0</td>
<td>30</td>
<td>1228</td>
<td>2.193</td>
<td>46.7</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>1532</td>
<td>2.193</td>
<td>55.5</td>
</tr>
<tr>
<td>60</td>
<td>30</td>
<td>1836</td>
<td>2.193</td>
<td>43.1</td>
</tr>
<tr>
<td>120</td>
<td>30</td>
<td>2500</td>
<td>2.193</td>
<td>7.9</td>
</tr>
<tr>
<td>120</td>
<td>20</td>
<td>2090</td>
<td>1.462</td>
<td>83.3</td>
</tr>
<tr>
<td>240</td>
<td>20</td>
<td>3742</td>
<td>1.462</td>
<td>-36.0</td>
</tr>
<tr>
<td>240</td>
<td>0</td>
<td>2923</td>
<td>0.0</td>
<td>-7.4</td>
</tr>
</tbody>
</table>

DIF1 % = Percentage difference between the longitudinal reinforcement computed by the direct design method and the other Codes of Practice.

DIF2 % = Percentage difference between the transverse reinforcement computed by the direct design method and the other Codes of Practice.

### 4. CONCLUSIONS

The results obtained in this paper lead to the following conclusions:

1. The direct design method can be applied to beams subjected to combined bending, torsion and shear after converting its cross-section to four outer plates subjected to in-plane stresses lumped from the applied loads.

2. The thickness of the plates is suggested to be twice the cover.

3. The rules set for the direct design method in this paper provide either an optimum reinforcement or a close upper bound to the minimum reinforcement. These rules will ensure that the yield criteria are nowhere violated, and that a state of yield will exist in most portions of the plate, sufficient to convert it into mechanism at failure.

4. The method of design developed in this study results in more economical reinforcement in comparison to the BS8110 and ACI codes of practice.

### REFERENCES


