CONNECTIVITY IN VEHICULAR NETWORK WITH MULTI-LANE AND EXPONENTIAL INTER-VEHICLE SPACING

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مُسْتَخْلَص

الاتصال اللاسلكي أمر في غاية الأهمية في شبكات السيارات لتبادل معلومات السلامة. كل البحوث السابقة قامت بحساب الاتصال بافتراض أن الشبكة عبارة عن بعد واحد تتوزع فيه السيارات توزيع منتظم في خط لانهائي وهو افتراض لا يمثل الوضع الطبيعي. في هذه الورقة يتم حساب الاتصال لشبكات السيارات في الطرق بمسار واحد وطرق بعدة مسارات. المسافة بين السيارات تتبع للتوزيع الأسي والشبكة تكون ذات بعد واحد أو بعدين حسب طبيعة الشارع. لمدى اتصال R وجد أن كثافة 7 سيارات لكل R أو أكثر تكون كافية للحصول على اتصال عالي داخل المدن بينما في الطرق السريعة يجب أن تكون الكثافة 12 سيارة لكل R على الأقل. الطرق مع العديد من المسارات تظهر متوسط اتصال أعلى ولكن الاحتمال أقل من الربط الكامل. تم كذلك دراسة أثر معدل الانتشار على الاتصال ووجد أن لمعدل انتشار 50-100% وكثافة 10 سيارات لكل Rيكون احتمال الربط الكامل 80% وتنخفض إلى 10% لمعدلات انتشار 5-20%.

Abstract

Wireless connectivity is essential for vehicle ad hoc (VANET) communications and it is vital for safety applications. Previous work considered the connectivity of a linear finite network where cars are uniformly distributed in an infinite single dimension which is not realistic. In this paper the connectivity of vehicle network is calculated for single and multi-lane roads. The spaces between cars on the road are considered to follow the exponential distribution in a two dimensional topology. The probability of full connectivity and average network connectivity of VANET with single and multi-lane roads is analysed. For communication range R, car densities of 7cars/R or more were found sufficient to provide high connectivity within the city compared to 12cars/R for highways. Roads with several lanes show higher average connectivity but lower probability of full connectivity. We also investigated the effect of penetration rate on connectivity and found that while penetration rates of 50-100%, 10cars/R provide 80% probability of full connectivity, this reduces to 10% for 5-20% penetration.

Keywords: *Connectivity, one-dimensional, vehicle ad hoc networks.*

1 INTRODUCTION

Vehicle Ad hoc Networks (VANETs) are a unique type of Mobile Ad hoc Networks (MANETs). In VANET the cars are constrained in a topology limited by the geometry of the road compared to the two dimensional topology of MANET. This constraint limits the connectivity of VANET since a multi-hop route may exist between two cars only if there are cars in the space between them whereas in MANET other routes may exist. A network is said to be fully connected if there exists a route between any two cars in the network regardless of the number of intermediate hops. The probability of full connectivity of

VANET and MANET has been considered in several papers for large (infinite) networks and was found to improve as the car density and/or transmission range increase [1-3]. In [4] the authors considered the connectivity of a linear finite network. They assumed the cars are uniformly distributed in a single dimension and obtained a closed form expression for the probability of full connectivity of VANET ($P_N(d)$) as:

$$P_N(d) = \begin{cases} 0, & N < d-1 \\ \left(1-\left(1-\frac{1}{d}\right)^N\right)^{N+1}, & N \geq d-1 \end{cases} \quad (1)$$

Where *N* is the number of cars and *d* is the network size. This expression applies for a single lane road, however in general the spaces between cars on the road follow the exponential distribution [1, 4]. Moreover roads typically have several lanes therefore the network can have two dimensions one much larger than the other.

In this paper we investigate the connectivity of VANET with single and multi-lane roads with the interspacing distances following the exponential distribution. The rest of the paper is organised as follows: In the next section a proposed connectivity model is introduced. The results from simulation are presented and analysed in Section 3 then Section 4 concludes the paper.

2 CONNECTIIVITY MODEL

We now introduce our model. Two cars *i* and *j* are connected if the distance ($X_{i,j}$) between them is less than or equal to the communication range (*R*) or there exists a set (*s*) of cars between *i* and *j* such that:

$$X_{i,a} \leq R, X_{b,j} \leq R \quad a, b \in S \quad (2)$$

Where *s*(*k*) is the element at index *k* in the set *s*. If there is no set that satisfies these conditions, then the two cars are not connected. To calculate the connectivity of the network shown in Figure 1, assume the distance between cars *i* and *i*-1 is greater than *R*, then these cars are not connected. However cars 0 to *i*-1 and *i* to *q* have interspacings less than *R* and are, hence, connected (dotted ellipses correspond to some of the connected sets).

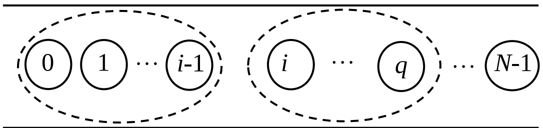

Figure 1: Network Layout

Besides the probability of full connectivity (*P*(*d*)), we will use another measure, average connectivity, which is the average number of cars each host can communicate with divided by the total number of cars. In a large and dynamic network such as VANET, it is very difficult to achieve full connectivity all the time. Several VANET attributes, e.g. safety, require the exchange of information between neighbouring cars. The average connectivity gives better insight than the probability of full connectivity. Let there be *n* sets in the network ($n \geq 1$) and within some set (*s*) there are q_s-i_s+1 cars, indexed i_s to q_s. Each car within this set can connect to T_s=q_s-i_s cars. The difference by 1 is because the car can always connect to itself. Therefore the connectivity of any car in *s* is given by:

$$C_{car} = \frac{T_s}{N-1} = \frac{q_s - i_s}{N-1} \quad (3)$$

Since at most, a car can connect to *N*-1 cars in a network of *N* cars, we define the connectivity of the network as the average connectivity of all the cars in the different sets given by:

$$C = \sum_{s=1}^{n} \sum_{j=i_s}^{q_s} \frac{T_s}{N(N-1)} = \sum_{s=1}^{n} \sum_{j=i_s}^{q_s} \frac{q_s - i_s}{N(N-1)} = \sum_{s=1}^{n} \frac{(q_s - i_s)(q_s - i_s + 1)}{N(N-1)} \quad (4)$$

The last expression follows since all cars within a single set will have equal connectivity. For multilane roads the interspacing for each lane is assumed to be independent from the other lanes with equal car density and the road width is negligible compared to the communication range. The probability of full connectivity ($P(d)$) is given by:

$$P(d) = \frac{No.\ of\ trials\ with\ fully\ connected\ network}{Total\ number\ of\ trials} \quad (5)$$

3 ANALATIC RESULTS

A. Single Lane Roads

Figure 2 compares the probability of full connectivity ($P(d)$) from our model with the mathematical model of equation (1) for R = 250m and networks of length 3R, 10R and 100R. Each point in our simulations is calculated from 1000 trials. As can be seen, both models show identical results for large networks despite the fact that equation (1) is for uniform distribution and the simulations use exponential distribution. This is due to the long distance of the road which makes the probability less affected by the distribution model used. The number of cars required to achieve full connectivity depends on the length of the road. As the road length increases, higher densities are required to achieve full connectivity. This is expected since with larger roads more hops are required to relay the messages and therefore the probability of connectivity is reduced. However we note a difference in the results between exponential and uniform distribution for small car densities. The reason for this difference is that cars tend to cluster in groups when exponential distribution is used whereas the cars are assumed to be distributed uniformly for the mathematical model. The uniform distribution thus is an idealised case and gives optimistic results. As the car density increases both distributions tend to produce similar results and the network becomes one large group. From the Figure we note that a car density of 7cars/R is sufficient to guarantee 95% probability of full connectivity for cars in short roads, correspond to city environments, compared to 10cars/R for a 100R road, corresponds to highways.

Figure 3 shows the average connectivity versus car density. Considering the 100R network, Figure 2 shows that for densities less than 5cars/R the probability of achieving full connectivity is approximately nil whereas we note from Figure 3 that on average, a car can communicate with 40% of the cars in the network. If the communication range is 1km as specified in the IEEE standard for VANET [5], this 40% corresponds approximately to a distance of 40km. In other words the car is able to receive information from cars up to 20km ahead and can relay it to cars 20km behind. This range is sufficient for most safety and traffic applications.

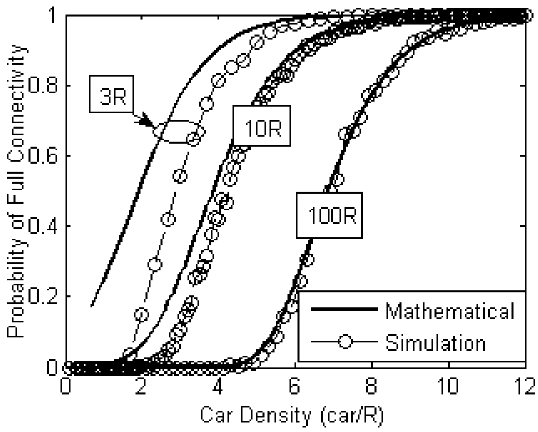

Figure 2: *P*(*d*) vs. Car Density, Single Lane

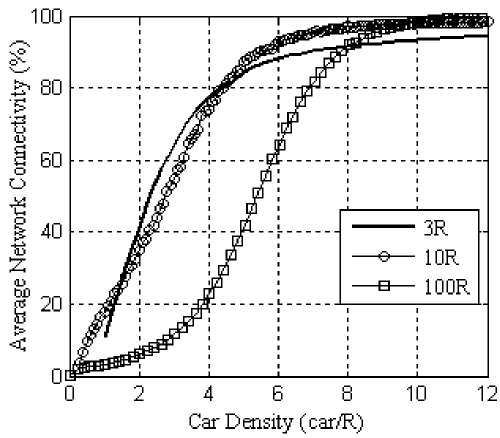

Figure 3: Connectivity vs. Car Density, Single Lane

The average connectivity in small networks is less sensitive to the network size compared to the probability of full connectivity. We observe from Figure 3 that for car densities of more than 4 and 6cars/R, 10R and 100R networks respectively have higher average connectivity than 3R. The reason behind this is the small number of cars in the 3R network. When there is only one isolated car the density is 10cars/R. This isolated car represents 33.3% of the network in a 3R network compared to 10% and 0.1% in 10R and 100R networks respectively.

B. Multilane Road

Now the case of more than 1 lane is investigated. Figures (4, 5) show the average connectivity and probability of full connectivity for 100R long roads that have 1, 2, 4 and 8 lanes. In multilane roads several cars can be at the same distance but in different lanes, thus several paths may exist between car pairs. This leads to a slight improvement in the average connectivity as the number of lanes increases as observed from Figure 4. However the probability of achieving full network connectivity is reduced as shown in Figure 5. The reason behind this reduction is that cars tend to cluster into groups with a large number of cars per groups. Due to this clustering it becomes more likely to have isolated groups or even isolated cars resulting in low *P*(*d*). Although the probability of full connectivity is low the network has high average connectivity because most of the cars in the network will be able to communicate with each other and the isolated cars form a small percentage thus having a small effect on the average but a large effect on the probability.

C. Penetration Rate

Penetration rate is the percentage of the cars having the necessary hardware and software to participate in VANET. In the previous section we assumed all cars on the road are equipped with communication devices to enable them communicate.

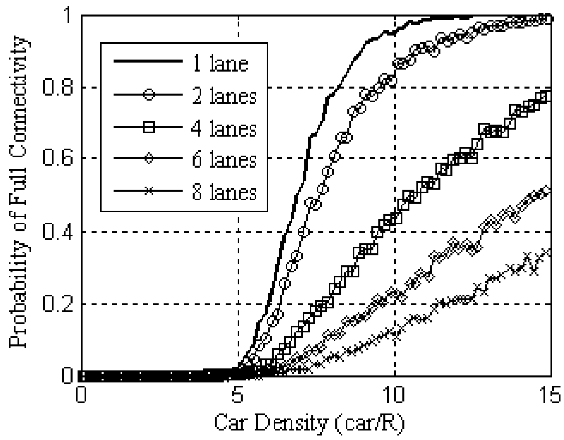

Figure 4: *P*(*d*) vs. Car Density, Milti Lane

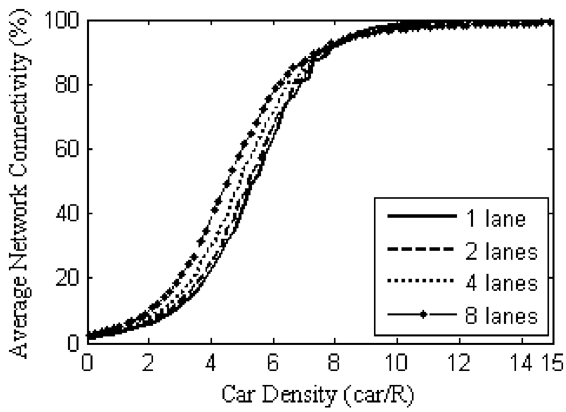

Figure 5: Connectivity vs. Car Density, Multi Lane

However the deployment of vehicular communication equipments will take time and therefore not all cars will be equipped at the beginning. We consider in our simulations penetration rates of 5%, 10%, 20%, 50%, and 70% and compare it to the previous results which represent 100% penetration. We assume a single lane road, 10R long. As seen from Figure 6, for penetration rates of 50% or more the probability of achieving full connectivity is greater than 80% for a density of 10cars/R.

However at low penetration rates (5-20%) this density is insufficient even to maintain 10% probability. We conclude from this that in the early stages of VANET deployment the network will provide little service for drivers on the road unless a large number of road side units are installed. Similar conclusion were reached in [6] and the authors suggest the provision of home-to-vehicle applications to encourage users to install the system.

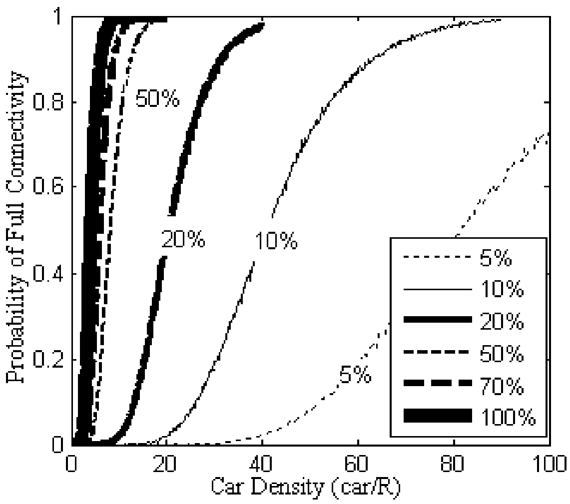

Figure 6: Effect of Penetration Rate on *P*(*d*)

4 CONCLUSION

The issue of connectivity in vehicular networks with multi-lane and exponential inter-vehicle spacing is investigated in this paper. From the results, it is concluded that for a 1 dimensional network 100R long to be fully connected the car density must be at least 12cars/R. Within the city, a density of 7cars/R is sufficient to guarantee a 95% probability of full connectivity for single lane roads. Roads with several lanes have higher average car connectivity but lower probability of full connectivity since cars tend to cluster into groups. This leads to isolated groups or even isolated cars thus resulting in very low probability of full connectivity. The isolated cars, however, have little effect on the average network connectivity. For penetration rates of 50-70% achieving 80% probability of full connectivity requires 10cars/R/lane while for penetration rates of 5-20% the probability for the same car density is less than 10%. Hence the use of an existing architecture is essential at the beginning of deployment.

REFERENCES

[1] Dousse, O., P. Thiran, and M. Hasler, "Connectivity in ad-hoc and hybrid networks," in *IEEE INFOCOM 2002*, 2002, pp. 1079-1088.

[2] Foh, C. H., G. Liu, B. S. Lee, B.-C. Seet, K.-J. Wong, and C. P. Fu, "Network Connectivity of One-Dimensional MANETs with Random Waypoint Movement," *IEEE Communications Letters,* vol. 9, pp. 31-33, 2005.

[3] Ghasemi A. and S. Nader-Esfahani, "Exact Probability of Connectivity in One-Dimensional Ad Hoc Wireless Networks," *IEEE Communications Letters,* vol. 10, pp. 251-253, Apr 2006.

[4] Foh C. H. and B. S. Lee, "A Closed Form Network Connectivity Formula for One-Dimensional MANETs," in *IEEE International Conference on Communications*, June 2004, pp. 3739-3742.

[5] "IEEE Draft P802.11p/D2.0, November 2006."

[6] Matheus, K., R. Morich, and A. Lübke, "Economic Background of Car-to-Car Communication," *http://www.network–on–wheels.de/documents .html*, 2004.